MATH 54, 2nd mock midterm test.

Name
Student ID #

All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit. Partial credit **may** be given but only for significant progress towards a solution. Show all relevant work in logical sequence and indicate all answers clearly. Cross out all work you do not wish considered. 1 page of notes is allowed. Books and electronic devices are not allowed during the test.

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1. Is the linear system

$$\begin{aligned} x_1 + x_2 &= 0 \\ x_1 - 3x_2 &= 0 \\ x_1 + 2x_2 &= 1 \end{aligned}$$

solvable? What is its least-squares solution? Is it unique?

2. Let A be an $m \times n$ matrix, and let B be an $n \times p$ matrix such that $AB = 0$. Show that $\operatorname{rank} A + \operatorname{rank} B \leq n$.

3. Show that the formula

$$\langle f, g \rangle := \frac{1}{2\pi} \int_0^{2\pi} f(t)g(t)dt$$

defines an inner product on $C[0, 2\pi]$, the vector space of all continuous function on the interval $[0, 2\pi]$. Let $W := \operatorname{span}\{1, \sin t, \cos t\}$. What is the orthogonal projection of the function $f(t) = t$ onto W?

4. A linear map $T : \mathbb{P}_2 \to \mathbb{P}_3$ is defined as $T : f(t) \mapsto (t^2 f(t))'$. Find a basis for $\ker(T)$.

5. Given

$$A := \begin{bmatrix} 1 & 2 \\ 1 & 0 \end{bmatrix},$$

compute A^{1000}.