

MATH 54, mock midterm test.

Name

Student ID #

All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit. Partial credit **may** be given but only for significant progress towards a solution. Show all relevant work in logical sequence and indicate all answers clearly. Cross out all work you do not wish considered. 1 page of notes is allowed. Books and electronic devices are not allowed during the test.

1. Let

$$A := \begin{bmatrix} 2 & 3 & 4 & 0 \\ -2 & -3 & -4 & 4 \\ 2 & 1 & 2 & -2 \\ -2 & -1 & -2 & 6 \end{bmatrix} \quad \text{and} \quad \vec{v} := \begin{bmatrix} 2 \\ -2 \\ 2 \\ 2 \end{bmatrix}.$$

Is the linear system $A\vec{x} = \vec{v}$ solvable?

2. Given the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & -2 \\ 0 & 1 & 1 \end{bmatrix},$$

(a) compute its determinant. (b) Is the matrix $A^T A$ invertible?

3. Are the matrices

$$A := \begin{bmatrix} 1 & 2 & 3 \\ -1 & 1 & -2 \\ 0 & 1 & 1 \end{bmatrix} \quad \text{and} \quad B := \begin{bmatrix} 2 & 1 & 0 \\ -1 & 1 & 1 \\ 3 & 2 & 1 \end{bmatrix}$$

row equivalent?

4. Mark each of the following true or false. Provide short explanations.

- The nonzero rows of a matrix form a basis for its row space.
- Elementary row operations on a matrix can change its null space.
- The nonpivot columns of a matrix are always linearly dependent.
- If A is an $m \times n$ matrix and the linear transformation $\vec{x} \mapsto A\vec{x}$ is onto, then $\text{rank } A = m$.