1. Let $\nu$, $\nu_1$, $\nu_2$ be signed measures and let $\mu$ be a positive measure on $(X, \mathcal{M})$. Prove the following statements:

(a) If $\nu_1 \perp \nu$ and $\nu_2 \perp \nu$, then $\nu_1 + \nu_2 \perp \nu$.
(b) If $\nu_1 \ll \mu$ and $\nu_2 \ll \mu$, then $\nu_1 + \nu_2 \ll \mu$.
(c) $\nu_1 \perp \nu_2$ implies $|\nu_1| \perp |\nu_2|$.
(d) $\nu \ll |\nu|$.
(e) If $\nu \perp \mu$ and $\nu \ll \mu$, then $\nu = 0$.

2. Will the definition of a signed measure (see p. 85) change if we replace absolute convergence by convergence in its 3rd requirement?

3. Suppose that $\{g_n\}$ is a sequence of positive continuous functions on $I = [0, 1]$, that $\mu$ is a positive Borel measure on $I$, and that

(a) $\lim_{n \to \infty} g_n(x) = 0$ m.a.e.;
(b) $\int_I g_n dm = 1$ for all $n$;
(c) $\lim_{n \to \infty} \int_I f g_n dm = \int_I f d\mu$ for every $f \in C(I)$.

Does it follow that $\mu \perp m$? Here $m$ denotes Lebesgue measure, as usual, and $C(I)$ denotes the space of all continuous functions on $I$.

4. Let $\mu$, $\nu$ be $\sigma$-finite measures on $(X, \mathcal{M})$ with $\nu \ll \mu$, and let $\lambda = \nu + \mu$. If $f = d\nu/d\lambda$, then $0 \leq f < 1$ m.a.e. and $d\nu/d\mu = f/(1-f)$.

5. Construct an example to show that Theorem 3.5 may fail if $\nu$ is not finite.