1. Given five points in a plane, no three of which lie on a straight line, show that some four of these points form the vertices of a convex quadrilateral.

2. Let \( S \) be the set of all numbers of the form \( 2^m3^n \), where \( m \) and \( n \) are integers, and let \( P \) be the set of all positive real numbers. Is \( S \) dense in \( P \)?

3. Let \( \alpha \) and \( \beta \) be given positive real numbers, with \( \alpha < \beta \). If two points are selected at random from a straight line segment of length \( \beta \), what is the probability that the distance between them is at least \( \alpha \)?

4. Prove that there is a constant \( K \) such that the following inequality holds for any sequence of positive numbers \( a_1, a_2, a_3, \ldots \):

\[
\sum_{n=1}^{\infty} \frac{n}{a_1 + a_2 + \cdots + a_n} \leq K \sum_{n=1}^{\infty} \frac{1}{a_n}.
\]

5. Let \( P_1, P_2, \ldots \) be a sequence of distinct points which is dense in the interval \((0, 1)\). The points \( P_1, P_2, \ldots, P_{n-1} \) decompose the interval into \( n \) parts, and \( P_n \) decomposes one of these into two parts. Let \( a_n \) and \( b_n \) be the lengths of these two intervals. Prove that

\[
\sum_{n=1}^{\infty} a_n b_n (a_n + b_n) = 1/3.
\]

6. If \( A \) and \( B \) are square matrices of the same size such that \( ABAB = 0 \), does it follow that \( BABA = 0 \)?

7. Let \( S \) be a set of three, not necessarily distinct, positive integers. Show that one can transform \( S \) into a set containing 0 by a finite number of applications of the following rule: Select two of the three integers, say \( x \) and \( y \), where \( x \leq y \) and replace them with \( 2x \) and \( y - x \).

8. Let

\[
\begin{array}{ccc}
  a_{1,1} & a_{1,2} & a_{1,3} & \cdots \\
  a_{2,1} & a_{2,2} & a_{2,3} & \cdots \\
  a_{3,1} & a_{3,2} & a_{3,3} & \cdots \\
  \vdots & \vdots & \vdots & \ddots
\end{array}
\]

be a doubly infinite array of positive integers, and suppose each positive integer appears exactly eight times in the array. Prove that \( a_{m,n} > mn \) for some pair of positive integers \((m,n)\).