MATH 128B, 1st midterm test, Wed, Feb 23rd.

Name
Student ID #

All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit. Partial credit **may** be given but only for significant progress towards a solution. Show all relevant work in logical sequence and indicate all answers clearly. Cross out all work you do not wish considered. Books and notes are allowed. Calculators and computers are not allowed during the test.

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1. (5pts total; 1pt for each subitem)
Suppose we invert a random diagonal matrix D of size 3000×3000 in MATLAB in five different ways (D was not specified as being sparse):

```
inv(D)

D^(-1)

diag(1./diag(D))

D\eye(length(D))

for k=1:length(D), D(k,k)=1/D(k,k); end
```

Identify which two methods executed in less than 1 sec and which three in more than 2 min.

1. solution continued

2. (10pts total, 2 pts for each subitem)
Consider the matrix

$$A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}$$

Determine its

1. eigenvalues and eigenvectors
2. spectral radius
3. 2-norm
4. ∞-norm
5. condition number for the ∞-norm

2. solution continued

3. (10pts total, 5 pts for each part)
Is Jacobi iteration guaranteed to converge for an upper triangular matrix with nonzero diagonal entries? Explain. Give an upper bound on the number of iterations.

3. solution continued

4. (10pts)
Apply the conjugate gradient algorithm to solve the 2×2 system $Ix = 0$ with

$$x^{(0)} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}, \qquad v_0 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \qquad v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

until reaching the solution. Draw a picture in the xy-plane showing succesive approximations to the solution and minimization directions.

4. solution continued