Homework # 9, due Fri, Apr 15th.

1. Use program 10.4 from Mathews and Fink to compute approximations for the harmonic function \( u(x, y) \) in the rectangle \( R = \{ (x, y) : 0 \leq x \leq 1.5, \ 0 \leq y \leq 1.5 \} \); use \( h = 0.5 \). The boundary values are

\[
\begin{align*}
    u(x, 0) &= x^4, \quad u(x, 1.5) = x^4 - 13.5x^2 + 5.0625 \quad 0 \leq x \leq 1.5 \\
    u(0, y) &= y^4, \quad u(1.5, y) = y^4 - 13.5y^2 + 5.0625 \quad 0 \leq y \leq 1.5.
\end{align*}
\]

Use the \texttt{surf} command to plot your approximation and compare it with the exact solution \( u(x, y) = x^4 - 6x^2y^2 + y^4. \)

2. Consider a knot sequence \( t := (0, 3, 5, 5.7) \). Write a MATLAB script that generates the following plots on the interval \([-1, 6]\): (i) \( B_{1,3,t} \), (ii) \( \omega_{1,3}B_{1,2,t} \), (iii) \( (1 - \omega_{2,3})B_{2,2,t} \). Comment on the significance of this plot.

3. Use the de Boor-Fix dual functionals \( \lambda_{jk} \) in order to prove the following property of splines: if \( f \) is a linear combination of B-splines of order \( k \) and it vanishes outside an interval \([t_{j+1}, t_{j+k}]\) for some \( j \), then \( f \) is zero everywhere.

   Also show that this result is tight, i.e., a spline \( f \) that vanishes outside an interval of the form \([t_j, t_{j+k}]\) need not be zero everywhere.