Homework # 9, due Fri, Apr 15th.

1. Use program 10.4 from Mathews and Fink to compute approximations for the harmonic function $u(x, y)$ in the rectangle $R = \{(x, y) : 0 \le x \le 1.5,\ 0 \le y \le 1.5\}$; use $h = 0.5$. The boundary values are

$$\begin{aligned} u(x,0) &= x^4, \quad u(x,1.5) = x^4 - 13.5x^2 + 5.0625 \quad 0 \le x \le 1.5 \\ u(0,y) &= y^4, \quad u(1.5,y) = y^4 - 13.5y^2 + 5.0625 \quad 0 \le y \le 1.5. \end{aligned}$$

Use the `surf` command to plot your approximation and compare it with the exact solution $u(x, y) = x^4 - 6x^2y^2 + y^4$.

2. Consider a knot sequence $\mathbf{t} := (0, 3, 5, 5.7)$. Write a MATLAB script that generates the following plots on the interval $[-1, 6]$: (i) $B_{1,3,\mathbf{t}}$, (ii) $\omega_{1,3}B_{1,2,\mathbf{t}}$, (iii) $(1-\omega_{2,3})B_{2,2,\mathbf{t}}$. Comment on the significance of this plot.

3. Use the de Boor-Fix dual functionals λ_{jk} in order to prove the following property of splines: if f is a linear combination of B-splines of order k and it vanishes outside an interval $[t_{j+1}, t_{j+k}]$ for some j, then f is zero everywhere.

Also show that this result is tight, i.e., a spline f that vanishes outside an interval of the form $[t_j, t_{j+k}]$ need not be zero everywhere.