Homework # 8, due Mon, Mar 28th.

For all MATLAB problems, turn in your code (and MATLAB diaries when needed). Please do not submit code which simply reproduces Algorithm 11.2 (p. 656) and Algorithm 11.4 (p. 669) of Burden and Faires. Rather, read the algorithms (described on pages 648 and 668) and implement them directly into Matlab where they become 5-line programs.

1. Create a MATLAB function for the nonlinear shooting algorithm and run it with step size $h = 0.1$ to approximate the solution to the boundary value problem

$$y'' = -(y')^2 - y + \ln x, \qquad 1 \le x \le 2, \;\; y(1) = 0, \;\; y(2) = \ln 2.$$

Compare your results to the exact solution $y = \ln x$.

2. Same as in Problem #1 with the nonlinear finite difference method.

3. (a) Verify that $u(x,t) = \sin(n\pi x)\cos(cn\pi t)$ is a solution to the wave equation $u_{tt}(x,t) = c^2 u_{xx}(x,t)$ for any positive integer n.

(b) For the equation $u_{tt}(x,t) = 9u_{xx}(x,t)$, what relationship between h and k must occur to produce the difference equation

$$u_{i,j+1} = u_{i+1,j} + u_{i-1,j} - u_{i,j-1}?$$

4. Suppose that u has the form

$$u(x,y) = ax^2 + bxy + cy^2 + dx + ey + f,$$

where a through f are constant coefficients. Find a relationship among those coefficients that guarantees that (a) $u_{xx} + u_{yy} = 0$, (b) $u_{xx} + u_{yy} = -1$.