Homework # 7, due Wed, Mar 16th.

For all MATLAB problems, turn in your code (and MATLAB diaries when needed).

1. Reformulate the I.V.P. for the second-order ODE

$$2x''(t) - 5x'(t) - 3x(t) = 45e^{2t}, \qquad x(0) = 2, \quad x'(0) = 1,$$

as an I.V.P. for a system of two first-order equations. Use MATLAB implementation of the Runge-Kutta method for systems to solve it on the interval $[0, 2]$ with step size $h = 0.05$. Plot your approximation and the analytic solution

$$x(t) = 4e^{-t/2} + 7e^{3t} - 9e^{2t}$$

on the same coordinate system.

2. Create a MATLAB function that solves a B.V.P. by the linear shooting method with built-in Runge-Kutte I.V.P. solver. Use it with step sizes 0.1, 0.05 and 0.025 to solve the B.V.P.

$$x'' - (1/t)x' + (1/t^2)x = 1, \qquad x(0.5) = 1, \;\; x(4.5) = 2. \tag{1}$$

Graph all solutions together on the interval $[0.5, 4.5]$.

3. Same as in Problem 2 using the finite-difference method.

4. Find an exact solution to B.V.P. (1) and compare numerical solutions in the previous two problems to it. (The comparison criterion is up to you.) Which method, in your opinion, performs better? Discuss.