Homework # 4, due Wed, Feb 16th.

For all MATLAB problems, turn in your code (and MATLAB diaries when needed).

1. Show that if B is a singular matrix, then

$$\frac{1}{K(A)} \leq \frac{\|A-B\|}{\|A\|}$$

where $K(A)$ is the condition number of A.
Hint: There exists a vector x with $\|x\| = 1$ such that $Bx = 0$.

2. Create a MATLAB function that inputs a matrix A, vectors b and $x^{(0)}$ and a tolerance tol and finds an approximate solution to $Ax = b$ using the conjugate gradient method (without preconditioning). The algorithm should terminate after n steps; it should output an error message in case the desired precision was not reached. Run an example for a linear system of your choice.

3. Same as in #2, with an additional (input) matrix C used for preconditioning. Run an example for a linear system and for several matrices C of your choice. What are good and bad choices of C for your system? Discuss.

4. With the notation used in the lecture, prove that

$$\langle v^{(j)}, Ax^{(k)} - b\rangle = 0 \quad \text{whenever} \quad j < k$$

in the conjugate gradient method.