

## Homework # 2, due Wed, Feb 2nd.

1. Verify the the function

$$\|x\|_1 := \sum_{j=1}^n |x_j|$$

is a norm on  $\mathbb{C}^n$ . Prove that  $\|x\|_1 \geq \|x\|_2$  for all  $x \in \mathbb{C}^n$ .

2. For each of the following matrices, find its characteristic polynomial, the eigenvalues and an eigenvector for each eigenvalue, and the spectral radius, using MATLAB.

$$(a) \begin{bmatrix} 1 & 2 \\ 3 & 2 \end{bmatrix}, \quad (b) \begin{bmatrix} -2 & 3 \\ 3 & -2 \end{bmatrix}, \quad (c) \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & 2 & 2 & 3 \\ 0 & 0 & 3 & 2 \\ 0 & 0 & 0 & 4 \end{bmatrix}.$$

3. For each of the matrices in Question 2, find its  $\infty$ -norm and its 2-norm using MATLAB or by hand.

4. Show that if  $A$  is symmetric, then  $\|A\|_2 = \rho(A)$ .