1. Create a MATLAB routine that inputs the order of the spline space $k$, a knot sequence $t$, a new knot $x$, a coefficient sequence $a$ and outputs new sequences $\hat{t}$ and $\hat{a}$ obtained after inserting $x$ into the knot sequence, so that

$$\sum_j a(j) B_{j,k,t} = \sum_j \hat{a}(j) B_{j,k,\hat{t}}.$$

Run your program on the data $k = 4, t = [0, 0, 1, 1.5, 2, 2.1, 2.2, 2.3, 3], a = [1, -1, -0.5, 3, -1]$. If you have access to the Spline toolbox, use it only to check your results.

2. Create a MATLAB function for plotting the control polygon of a spline given its order $k$, knot sequence $t$ and a coefficient sequence $a$. Run it with the same data as in Problem 1.

3. Create a MATLAB function for inserting knots into control polygons. Experiment inserting knots into the control polygon from Problem 2 to achieve a control polygon closely resembling the spline itself.