Mock final exam, Math 128B

1. Find the limit $\lim_{n\to\infty} A^n v$ where
$$A = \begin{bmatrix} 2/3 & 6 & 10 \\ 0 & 1/2 & 1/2 \\ 0 & 1/3 & 1/2 \end{bmatrix}, \qquad v = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}.$$

2. Convert the equation $y''' = x^2 y'' - xy' + y$ into a system of linear ODEs, and perform one step of Heun's method with step site $h = 0.5$ using initial values
$$y(0) = 0, \;\; y'(0) = 1, \;\; y''(0) = -1.$$

3. Solve the boundary value problem
$$y'' = 2y' - y, \quad y(0) = 1, \quad y(1) = 2$$
by linear shooting.

4. Write a matrix equation that arises from the discretization of the boundary value problem
$$\begin{aligned} u_{xx} = u_{yy}, \qquad & u(x,0) = 0, \quad u(x,2) = 1, \quad x \in [0,2], \\ & u(0,y) = \sin \pi y + \frac{y}{2}, \quad u(2,y) = \frac{1}{2} - \frac{1}{2}\cos\frac{\pi y}{2}, \quad y \in [0,2], \end{aligned}$$
on the square $[0,2] \times [0,2]$ with mesh sizes $h = k = 0.5$.

5. Critique the following MATLAB routine for testing the Schoenberg-Whitney condition:

```
function [T, Tau, passed] = SchoenbergWhitney(t, tau)

N = length(tau);
k = length(t) - N;

T = sort(t);
Tau = sort(tau);
passArray = (T(1:N) < Tau) & (T(1+k:end) > Tau);
passed = (nnz(passArray) == N);

return
```

6. Given a spline of order $k = 3$ with coefficients $(1, -2, 1, -1, 3, 4)$ on the knot sequence $(0, 1, 3, 6, 8, 10, 11, 15, 16)$, find its coefficients after inserting the knot $x = 8$ (so as it becomes a double knot).

7. Prove that the definition of the third order cardinal spline via convolution $B_3 = B_1 * B_2$ agrees with its usual definition via the recurrence relation.