Math 128A, Fall 2016.

Homework 4, due Sep 28th.

Prob 1. Show that the updated submatrices arising in Gaussian elimination of a symmetric matrix are necessarily symmetric. Use this fact to conclude that the multiplicative complexity of Gaussian elimination of a symmetric matrix is $n^3/6 + O(n^2)$.

Prob 2. Prove Lemma 1 on p.40 of our main textbook.

Prob 3. Prove: if $\lim_{n\to\infty} A_n = A$ and A is invertible, then A_n is invertible for all sufficiently large n and

$$\lim_{n\to\infty} \|A_n^{-1} - A^{-1}\| = 0.$$

Prob 4. Let C be an approximate inverse for A so that the norm of the matrix $E := I - CA$ is strictly less than 1, and let z be a fixed vector.

(a) Prove that the function $x \mapsto Ex + z$ is Lipschitz with Lipschitz constant less than 1.

(b) Prove that the sequence (x_n) defined by $x_{n+1} := Ex_n + z$ converges to the same limit for any x_0.

(c) What is the limit of (x_n)?

Prob 5. Write a MATLAB routine that takes A, C, and z as inputs, checks the norm condition on E (using your favorite norm) and implements the iteration from Prob. 4(b). Print out results for three sets of inputs. Do not use diagonal or block-diagonal matrices.