Math 128A, Fall 2016.

Homework 3, due Sep 21st.

Prob 1. Let A be an $n \times n$-matrix. Given any vector norm in $\mathbb{R}^n$, prove that the problem

$$A\vec{x} = \vec{f}$$

is well-posed in that norm for all $\vec{f}$ if and only if A is non-singular.

Prob 2. Show that Gaussian elimination with pivoting applied to the system $A\vec{x} = \vec{f}$ of n linear equations in n unknown leads to finding matrices L, U, P, Q such that

$$A = PLUQ^T, \qquad \text{where}$$

U is upper-triangular, L is lower-triangular with 1s on the diagonal, and P and Q are permutation matrices.

Prob 3. In the setting of Prob. 2, express the right-hand obtained after Gaussian elimination with pivoting in terms of the original right-hand side $\vec{f}$ and the matrices L, U, P, Q. How can you read off the pivot positions from P and Q?

Prob 4. Can a sequence of nonsingular 3×3-matrices A_n converge but the sequence of solutions to the systems $A_n\vec{x} = \vec{f}$ diverge? (Note that $\vec{f}$ stays constant.) Supply numerical evidence for your answer, using Gaussian elimination in MATLAB.

Prob 5. Suppose you need to solve m problems $A\vec{x_j} = \vec{f_j}$, $j = 1, \ldots, m$, where A is a fixed nonsingular $n \times n$-matrix and the right-hand sides $\vec{f_j}$ are distinct. When is it more efficient, in terms of arithmetic complexity, to run m Gaussian eliminations, and when to invert the matrix A and multiply the inverse by the right-hand sides? How does the answer change if you consider multiplicative compexity instead?