

Math 128A, Fall 2016.

Homework 1, due Sep 7th.

Prob 1. Show that if $\|\cdot\|$ is a vector norm and A is a non-singular matrix, then $x \mapsto \|Ax\|$ is a(nother) vector norm. What happens if A is singular?

Prob 2. Prove that $A \mapsto \max_{i,j} |a_{ij}|$ is a norm on the space of matrices.

Prob 3. Is the norm from Prob. 2 induced by some vector norm?

Prob 4. Show that any matrix norm that satisfies the condition $\|AB\| \leq \|A\| \cdot \|B\|$ for any A and B also satisfies the conditions

$$\|I\| \geq 1 \quad \text{and} \quad \|A^{-1}\| \geq \frac{1}{\|A\|}$$

for any invertible matrix A .

Prob 5. Prove or disprove: the spectral radius $A \mapsto \rho(A)$ is a matrix norm.