Mock midterm.

1. Prove the following theorem (Fredholm's alternative): Let $A : V \to W$ be a linear map between two inner product spaces. Let b be any element in W. Then either $Ax = b$ has a solution for some $x \in V$ or there is a vector $w \in W$ wtih $A^*w = 0$ and $\langle b, w \rangle_W \neq 0$.

2. Use Parseval's theorem to evaluate the series

$$\sum_{n=1}^{\infty} \frac{1}{n^4}.$$

3. With ϕ the Haar scaling function, find its convolution with itself $\phi * \phi$.

4. Let ϕ and ψ be the Haar scaling function and the Haar wavelet, respectively, and let the spaces V_j and W_j be defined as in the lectures. Let a function f be given by

$$f(x) = \begin{cases} 10 & 0 \leq x < 1/4, \\ -4 & 1/4 \leq x < 1/2, \\ -1 & 1/2 \leq x < 3/4, \\ 7 & 3/4 \leq x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Express f in terms of the natural basis of V_2 and then decompose it into its V_0, W_0 and W_1 components.