MATH 118, mock final test.

1. Suppose $f \in L^2(\mathbb{R})$ is a real-valued, even function; show that $\widehat{f}$ is real-valued. Suppose that $f \in L^2(\mathbb{R})$ is a rela-valued, odd function; show that $\widehat{f}$ is purely imaginary.

2. Expand the function $f(x) = \cos x$ in a Fourier sine series on the interval $0 \leq x \leq \pi$.

3. Are the following filters (a) linear (b) time-invariant (c) causal?

1. $(L_1 f)(t) = f(-t) - \int_t^{2t} f(x)\mathrm{dx}$,
2. $(L_2 f)(t) := \int_t^{\infty} f(x)e^{-(x-t)^2}\mathrm{dx}$.

4. Suppose that $\{V_j : j \in \mathbb{Z}\}$ is a multiresolution analysis (MRA) with scaling function ϕ that is continuous and compactly supported.

- Find u_j, the orthogonal projection onto V_j of the box function B_1 (i.e., the characteristic function of the interval $[0, 1]$).
- If $\int_{\mathbb{R}} \phi(x)\mathrm{dx} = 0$, show that $\|B_1 - u_j\| \geq 1/2$ for sufficiently large j.
- Explain why this implies that $\int_{\mathbb{R}} \phi(x)\mathrm{dx}$ should not be zero if ϕ is a scaling function generating an MRA.

5. Let ϕ_2 denote the Daubechies-2 scaling function. Show that the function $\widetilde{\phi}(x) := \phi_2(\frac{3}{2} - x)$ is refinable with the mask obtained from the mask of ϕ_2 by reversing the order of coefficients. What is the corresponding equation for $\widetilde{\psi}(x) := \psi_2(\frac{3}{2} - x)$?

6 (hard). Verify that the function ϕ defined via its Fourier transform

$$\widehat{\phi}(\xi) := 2\sqrt{\frac{2}{\pi}} \frac{\sin^2(\xi/2)}{\xi^2 \sqrt{1 - \frac{2}{3}\sin^2(\xi/2)}}$$

has orthonormal shifts.

7. Suppose you are sampling a function periodic with period T. The function looks very smooth, but you suspect that it has discontinuities in its third derivative. You want to detect them using wavelets. Discuss the computational issues involved such as sampling rate, signal extension, levels of decomposition, wavelet system to use, and describe your detection procedure.