

## Homework # 7, due Fri, Nov 4th.

1. Let  $P(z) = (1/2) \sum_k p_k z^k$  and define

$$Q(z) := -z \overline{P(-z)}.$$

Show that  $Q(z) = (1/2) \sum_k (-1)^k \overline{p_{1-k}} z^k$ . What is the relevance of this formula?

2. Show that if a scaling function  $\phi$  with orthonormal shifts has compact support, then there is only a finite number of nonzero coefficients in its scaling relation.

3. Show that if the  $p_k$ 's in the scaling relation are all zero for  $k > N$  and  $k < 0$  and the iterates  $\phi_n$  in the Cascade algorithm converge to  $\phi$ , then  $\phi$  has compact support. Find the length of the support of  $\phi$  in terms on  $N$ .

4. Using MATLAB, run the Cascade algorithm for the mask  $P(z) = (1/2) \sum_{k=0}^3 p_k z^k$  where

$$p_0 = \frac{1 + \sqrt{3}}{4}, \quad p_1 = \frac{3 + \sqrt{3}}{4}, \quad p_2 = \frac{3 - \sqrt{3}}{4}, \quad p_3 = \frac{1 - \sqrt{3}}{4}.$$

Graph the resulting function. How smooth is it?