Homework # 6, due Fri, Oct 21st.

1. Let ϕ and ψ be the Haar scaling function and the Haar wavelet, respectively, and let the spaces V_j and W_j be defined as in the lectures. Let a function f be given by

$$f(x) = \begin{cases} -1 & 0 \le x < 1/4, \\ 4 & 1/4 \le x < 1/2, \\ 2 & 1/2 \le x < 3/4, \\ -3 & 3/4 \le x < 1, \\ 0 & \text{otherwise.} \end{cases}$$

Express f in terms of the natural basis of V_2 and then decompose it into its V_0, W_0 and W_1 components.

2. What is the dimension of the spaces W_n and V_n restricted to the interval $[0, 1]$?

3. Reconstruct a function $f \in V_3$ given these coefficients in its Haar decomposition:

$$a^{[1]} = [3/2,\ -1], \quad b^{[1]} = [-1,\ -3/2], \quad b^{[2]} = [-3/2,\ -3/2,\ -1/2,\ -1/2].$$

The first entry in each list corresponds to $k = 0$. Sketch f.

4. Consider the function $f(x) := x^2$ on $[0, 1]$. What dyadic level of discretization produces a piecewise-constant version f_d of f with relative error less than 0.1? The relative error is taken with respect to the $L^2[0, 1]$-norm, i.e., is equal to

$$\frac{\|f - f_d\|_{L^2[0,1]}}{\|f\|_{L^2[0,1]}}.$$