Homework # 2, due Fri, Sep 16th.

1. By using Gram-Schmidt, find an orthonormal basis for the subspace of $L^2[0,1]$ spanned by 1, x, x^2, x^3.

2. Find the $L^2[0,1]$-orthogonal projection of the function $\cos x$ onto the span of 1, x, x^2, x^3.

3. Suppose u_0 and u_1 are vectors in an inner product space V such that $\langle u_0, v\rangle = \langle u_1, v\rangle$ for all $v \in V$. Show that $u_0 = u_1$.

4. Suppose A in an $n \times n$ matrix with complex entries. Show that the following are equivalent.

(a) The rows of A form an orthonormal basis in $\mathbb{C}^n$.

(b) $AA^* = I$ (the identity matrix).

(c) $\|Ax\| = \|x\|$ for all $x \in \mathbb{C}^n$.