Homework # 1, due Fri, Sep 9th.

1. Define $\langle v, w\rangle$ for $v = (v_1, v_2)$ and $w = (w_1, w_2) \in \mathbb{C}^2$ as

$$\langle v, w\rangle = (\overline{w_1}, \overline{w_2}) \begin{pmatrix} 1 & 2 \\ 2 & 4 \end{pmatrix} \begin{pmatrix} v_1 \\ v_2 \end{pmatrix}.$$

Does $\langle \cdot, \cdot \rangle$ define an inner product?

2. For $n \in \mathbb{N}$, let

$$f_n(t) := \begin{cases} 1, & 0 \leq t \leq 1/n, \\ 0, & \text{otherwise.} \end{cases}$$

Show that $f_n \to 0$ in $L^2[0, 1]$. Show that f_n does not converge to zero uniformly on $[0, 1]$.

3. What is the orthogonal complement in $\mathbb{R}^3$ of the span of $(1, -2, 1)$?

4. Let $f(t) = 1$ on $[0, 1]$. What is the orthogonal complement of the span of f in $L^2[0, 1]$?

5. Suppose that f is a differentiable function that is orthogonal to cos in $L^2[0, \pi]$. Show that f' is orthogonal to sin in $L^2[0, \pi]$.