MATH 118, final test, Dec 13th.

Name
Student ID #

All the necessary work to justify an answer and all the necessary steps of a proof must be shown clearly to obtain full credit. Partial credit **may** be given but only for significant progress towards a solution. Show all relevant work in logical sequence and indicate all answers clearly. Cross out all work you do not wish considered. Books and notes are allowed. Calculators, computers, cell phones, pagers and similar devices are not allowed during the test.

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1. (10pts total) Expand the function $f(x) = e^{rx}$, $r > 0$, in a Fourier series valid for $-\pi \leq x \leq \pi$. What is the pointwise limit of this series in $\mathbb{R}$?

2. (8pts total) Suppose L is a linear time-invariant *delay* filter, i.e., its output begins t_0 time units later than its input arrives. What integral form must L have? Explain why.

3. (12pts total) For sufficiently large $\Omega > 0$, the sum

$$\sum_{j=-\infty}^{\infty} \frac{\sin^2(j\pi/\Omega)}{j\pi/\Omega} \cdot \frac{\sin(\Omega t - j\pi)}{\Omega t - j\pi}$$

can be simplified into one fraction involving the sinc function. Perform the simplification. For which values of Ω does it work?

4. (10pts total) Let $\{V_j\}$ denote the Haar multiresolution analysis. Reconstruct the function $g \in V_3$ from its coefficients

$$a^{[2]} = [1/2, 2, 5/2, -3/2], \quad b^{[2]} = [-3/2, -1, 1/2, -1/2].$$

The first entry in each list corresponds to $k = 0$, and we use the notation $g = v_j + w_j$, where

$$v_j = \sum_{k \in \mathbb{Z}} a_k^{[j]} \phi(2^j x - k), \quad w_j = \sum_{k \in \mathbb{Z}} b_k^{[j]} \psi(2^j x - k).$$

Sketch g.

5. (6pts total) Let ϕ be a refinable function with refinement mask $P(z) = \sum_k p_k z^k$. Does there always exist a refinable function with mask $\widetilde{P}(z) := \sum_k p_{-k} z^k$?

6. (14pts total) Show that a step of the wavelet decomposition algorithm based on a CQF preserves the l^2 norm of a signal. That is, let T_0 be convolution with the low-pass filter followed by downsampling and let T_1 be convolution with the high-pass filter followed by downsampling. Then

$$\|T_0 x\|^2 + \|T_1 x\|^2 = \|x\|^2 \quad \text{for all } x \in l^2.$$