MATH 104, FALL 2013.
Homework assignment # 7.

1. Let $a_1 = 1$ and $a_n = \sqrt{3a_{n-1} + 4}$. Prove that the sequence $\{a_n\}$ is bounded.

2. Suppose that $x_1 \in \mathbb{R}$ and $x_{n+1} = \sqrt{1 + x_n^2}$ for all $n \in \mathbb{N}$. Show that $\{x_n\}$ does not converge.

3. Suppose that $f_1(x) = x$ for $x \in \mathbb{R}$ and that $f_{n+1}(x) = (f_n(x))^2/2$ for $n \geq 1$. If $\lim_{n\to\infty} f_n(x)$ exists, what can the limit equal? For which x is the sequence $\{f_n(x)\}$ strictly increasing, constant, or strictly decreasing? Use this information to determine for which x the limit $\lim_{n\to\infty} f_n(x)$ exists and how it depends on x.

NB: The metric in Problems 1 - 3 above is the usual metric $|\cdot|$ on $\mathbb{R}$. Problem 3 is not compulsory and is included as a challenge problem / preview to sequences of functions.