1. Let \( a_1 = 1 \) and \( a_n = \sqrt{3a_{n-1} + 4} \). Prove that the sequence \( \{a_n\} \) is bounded.

2. Suppose that \( x_1 \in \mathbb{R} \) and \( x_{n+1} = \sqrt{1 + x_n^2} \) for all \( n \in \mathbb{N} \). Show that \( \{x_n\} \) does not converge.

3. Suppose that \( f_1(x) = x \) for \( x \in \mathbb{R} \) and that \( f_{n+1}(x) = (f_n(x))^2/2 \) for \( n \geq 1 \). If \( \lim_{n \to \infty} f_n(x) \) exists, what can the limit equal? For which \( x \) is the sequence \( \{f_n(x)\} \) strictly increasing, constant, or strictly decreasing? Use this information to determine for which \( x \) the limit \( \lim_{n \to \infty} f_n(x) \) exists and how it depends on \( x \).

**NB:** The metric in Problems 1 - 3 above is the usual metric \( | \cdot | \) on \( \mathbb{R} \). Problem 3 is not compulsory and is included as a challenge problem / preview to sequences of functions.