1. Give an example of a function $f$ such that $|f|$ is Riemann integrable on $[0,1]$ but $f$ is not Riemann integrable on $[0,1]$.

2. Evaluate 
   \[ \frac{d}{dx} \int_0^x f(x,t) dt \]
   provided that $\frac{\partial}{\partial x} f(x,t)$ exists and is Riemann integrable (in $t$) for all $x \in \mathbb{R}$. Apply the result to the function $f(x,t) = e^{t+x} \sin(tx)$.

3. Let $f(x) = x^2$ and let $P_n$ be the partition of $[1,3]$ into $n$ subintervals of equal length. Compute formulas for $L(f,P_n)$ and $U(f,P_n)$ in terms of $n$. Verify that they have the same limit as $n \to \infty$. Determine how large $n$ must be to ensure that $U(f,P_n)$ is within 0.01 of \( \int_1^3 f(x) dx \).

4. Suppose $\alpha$ is monotonically increasing on $[a,b]$, $g$ is continuous on $[a,b]$, and $g(x) = G'(x)$ for $a \leq x \leq b$. Prove that
   \[ \int_a^b \alpha(x)g(x)dx = G(b)\alpha(b) - G(a)\alpha(a) - \int_a^b Gd\alpha. \]