MATH 104, FALL 2013.
Homework assignment # 14.

1. Give an example of a function f such that $|f|$ is Riemann integrable on $[0,1]$ but f is not Riemann integrable on $[0,1]$.

2. Evaluate

$$\frac{d}{dx}\int_0^x f(x,t)dt$$

provided that $\frac{\partial}{\partial x}f(x,t)$ exists and is Riemann integrable (in t) for all $x \in \mathbb{R}$. Apply the result to the function $f(x,t) = e^{t+x}\sin(tx)$.

3. Let $f(x) = x^2$ and let P_n be the partition of $[1,3]$ into n subintervales of equal length. Compute formulas for $L(f,P_n)$ and $U(f,P_n)$ in terms of n. Verify that they have the same limit as $n \to \infty$. Determine how large n must be to ensure that $U(f,P_n)$ is within 0.01 of $\int_1^3 f(x)dx$.

4. Suppose α is monotonically increasing on $[a,b]$, g is continuous on $[a,b]$, and $g(x) = G'(x)$ for $a \le x \le b$. Prove that

$$\int_a^b \alpha(x)g(x)dx = G(b)\alpha(b) - G(a)\alpha(a) - \int_a^b G d\alpha.$$