

MATH 104, FALL 2013.
Homework assignment # 13.

1. The following inductive argument fails to prove that $\frac{d}{dx}x^n = nx^{n-1}$ for nonnegative integers n . Explain the error and correct the proof.

Induction basis ($n = 0$): $\lim_{h \rightarrow 0} \frac{1-1}{h} = 0$.

Inductive step ($n > 0$): Using the induction hypothesis and the product rule, we get

$$\frac{d}{dx}x^n = \frac{d}{dx}x \cdot x^{n-1} = x(n-1)x^{n-2} + 1 \cdot x^{n-1} = nx^{n-1}.$$

2. Suppose that $|f(x)| \leq x^2 + x^4$ for all x . Prove that $f'(0)$ exists.

3. Let f be differentiable and such that $f'(x) < 1$ for all $x \in \mathbb{R}$. Prove that f has at most one fixed point. (Recall that x is a fixed point of f if $f(x) = x$.)

4. The volume of a ball of radius r is $\frac{4}{3}\pi r^3$. Suppose the air is escaping from a ball at the rate of 36 cubic inches per second. How fast is the radius of the ball decreasing at the moment when the radius is 6 inches?