

**MATH 104, FALL 2013.**  
**Homework assignment # 1.**

1. Prove that  $\sqrt{2} + \sqrt{3}$  is irrational.
2. Let  $A$  and  $B$  be two nonempty sets of reals that are bounded above and below. Denote

$$A - B := \{a - b : a \in A, b \in B\}.$$

Express, with proof,  $\sup(A - B)$ ,  $\inf(A - B)$  in terms of  $\sup A$ ,  $\inf A$ ,  $\sup B$ ,  $\inf B$ .

3. Prove that the set of  $2 \times 2$ -matrices of the form

$$\begin{bmatrix} a & b \\ -b & a \end{bmatrix} : a, b \in \mathbb{R}$$

with the usual matrix addition and multiplication is a field.

4. Can the field from Problem 3 be turned into an ordered field by an appropriate choice of order? Justify your answer.