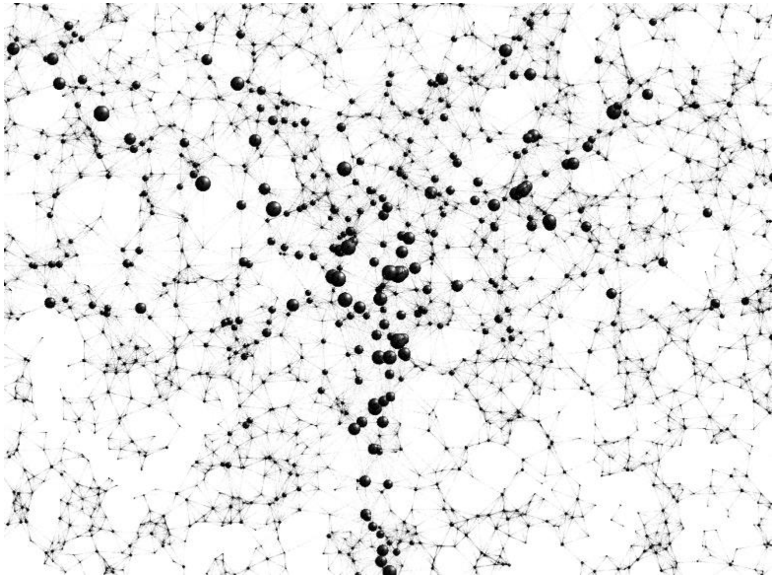
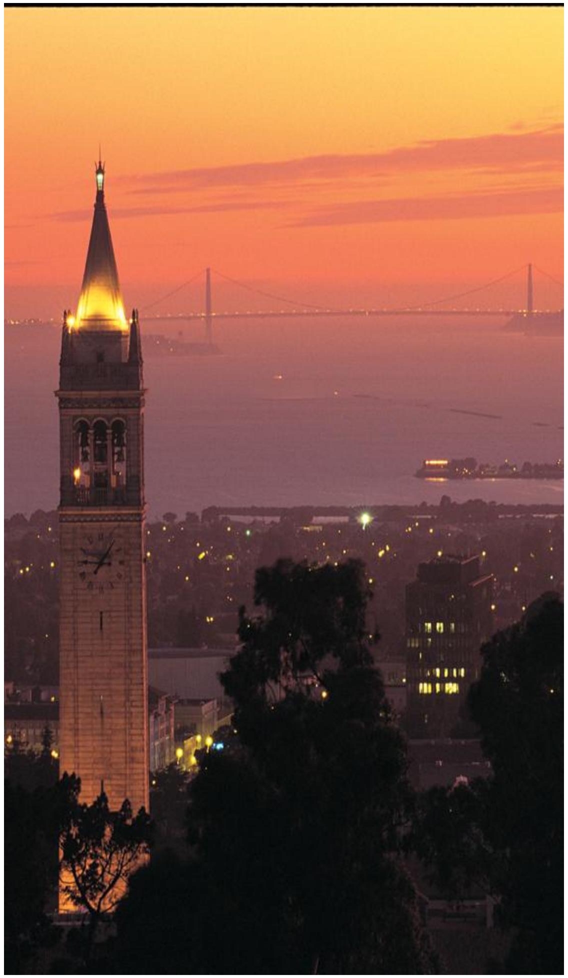
Modern **L**ow-Complexity **C**apacity-Achieving **C**odes For **N**etwork **C**ommunication

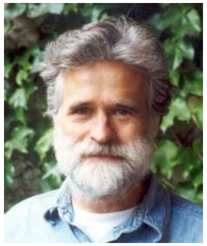
Naveen Goela



May 2, 2013

Thank You



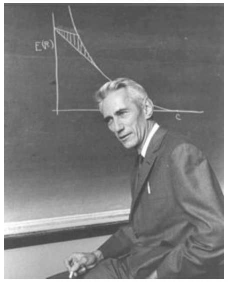





Information **T**heory

Information Theory

What is the Abstraction of Information ?



Information Theory

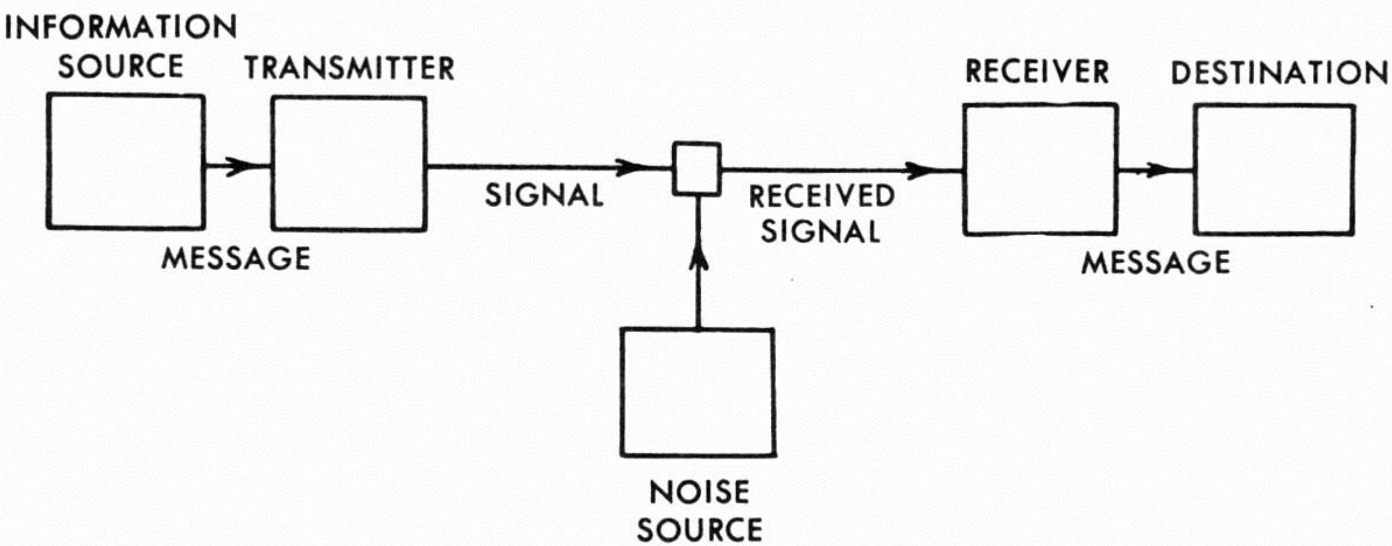
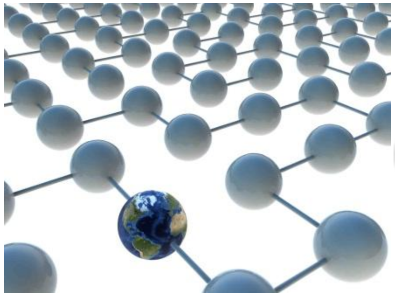


Fig. 1. — Schematic diagram of a general communication system.

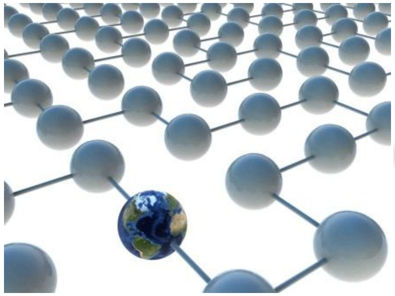
Modern Information Theory

Networks ?

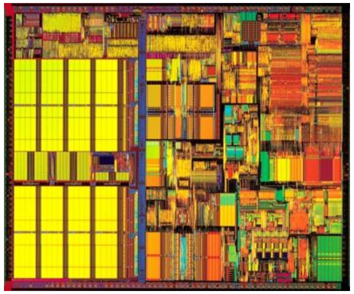


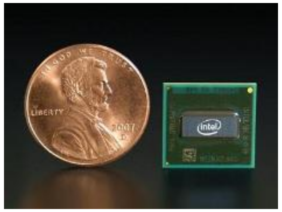
Modern Information Theory

Networks ?

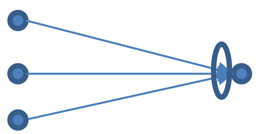
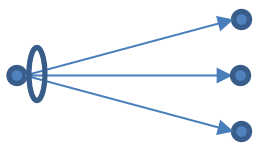
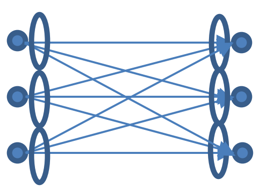
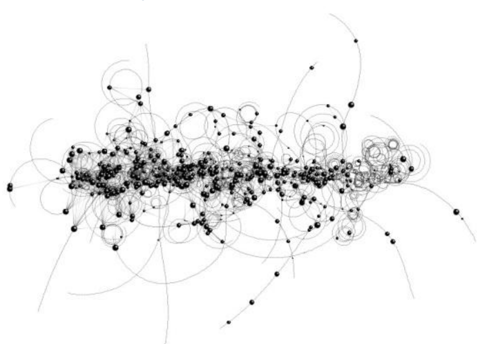


Low-complexity ?

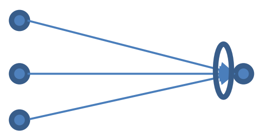
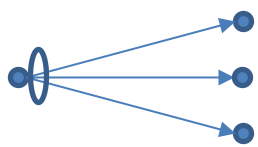
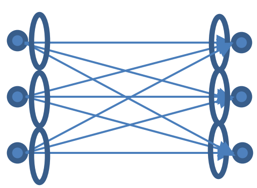
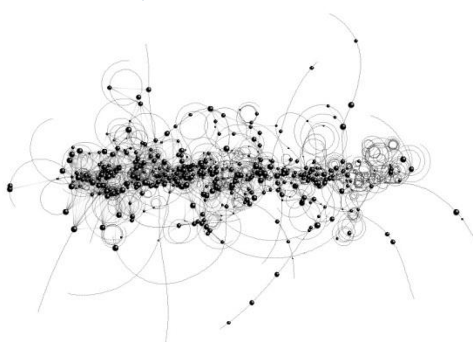




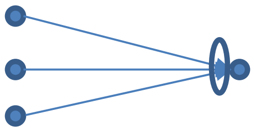
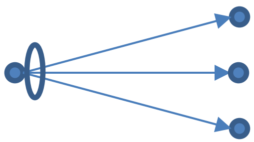
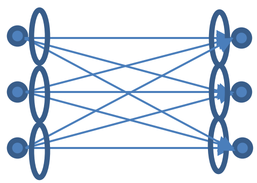
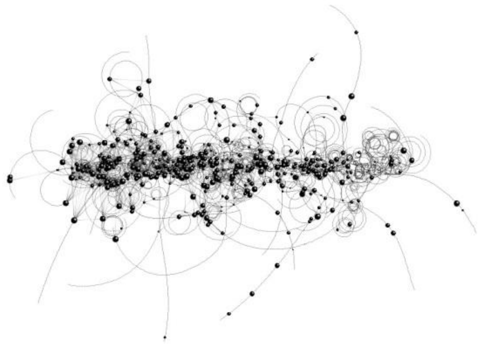
Information Theory of Networks

Topology	Capacity	Codes (Practical)
		
		
		
		
		

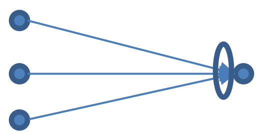
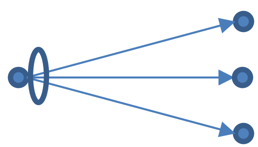
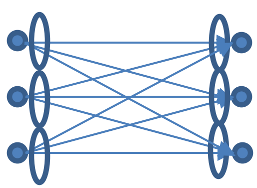
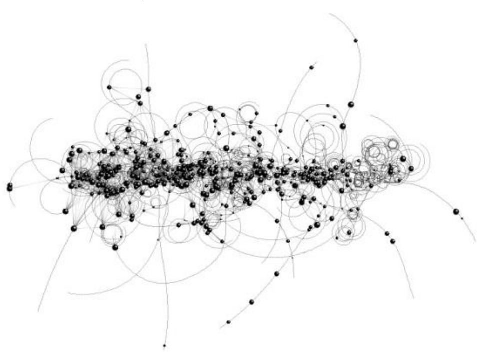
Information Theory of Networks

Topology	Capacity	Codes (Practical)
		
		
		
		
		

Information Theory of Networks

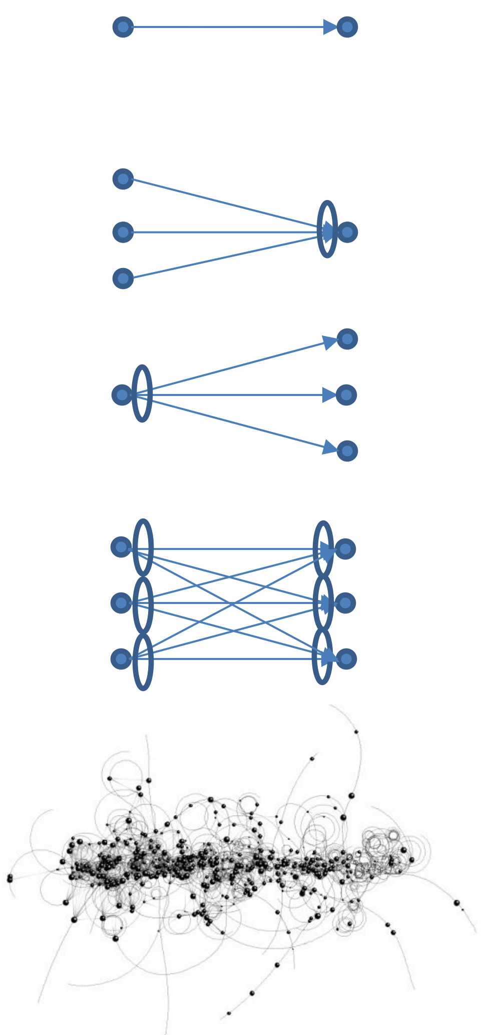
Topology	Capacity	Codes (Practical)
	✓	
	✓	
		
		
		

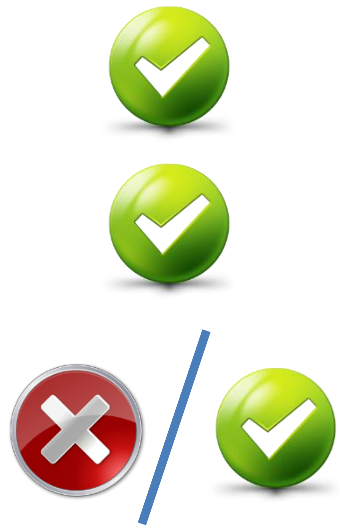
Information Theory of Networks

Topology	Capacity	Codes (Practical)
		
		
		
		
		

Information Theory of Networks

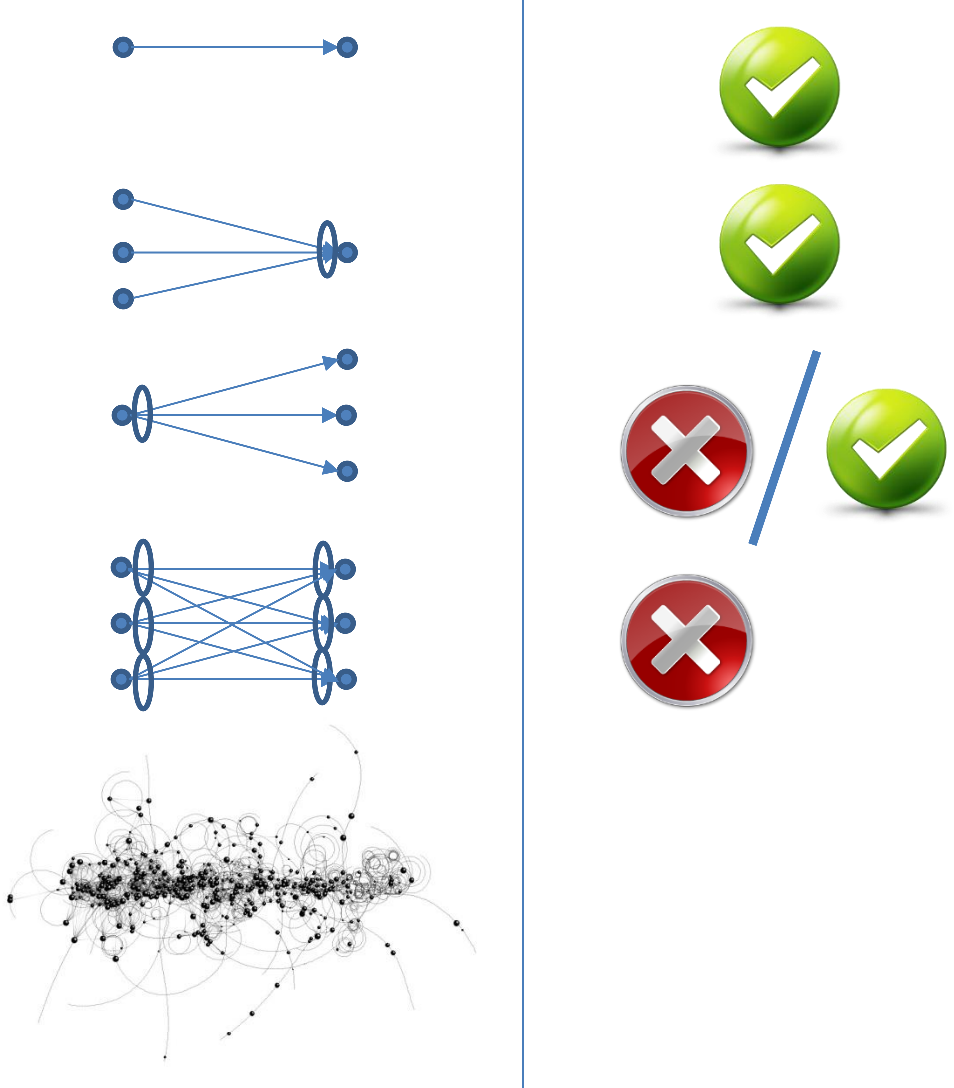
Topology	Capacity	Codes (Practical)



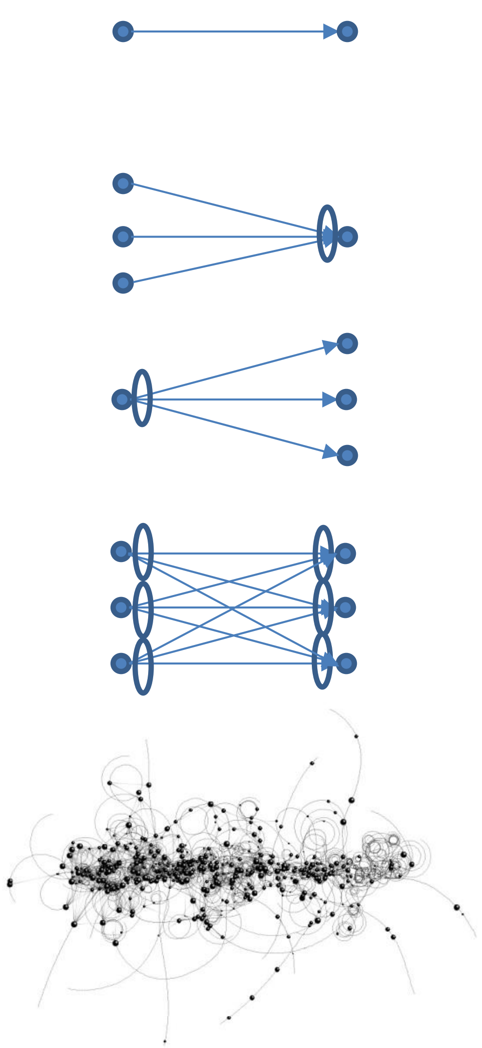


Information Theory of Networks

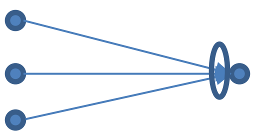
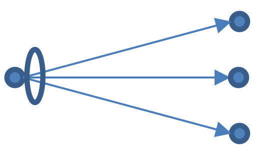
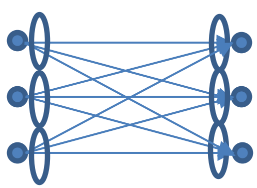
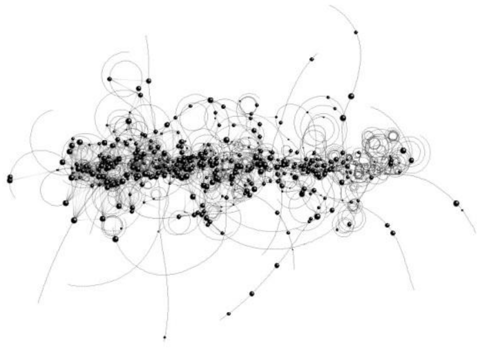
Topology	Capacity	Codes (Practical)



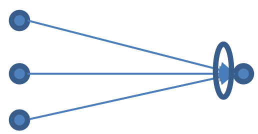
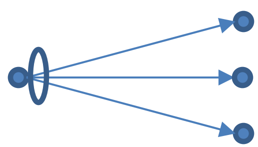
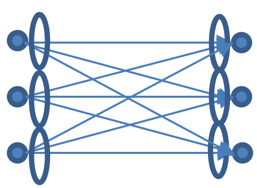
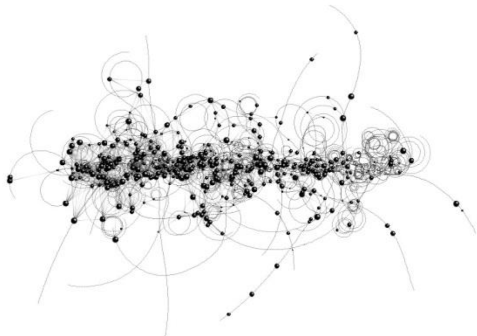
Information Theory of Networks

Topology	Capacity	Codes (Practical)
		

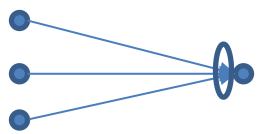
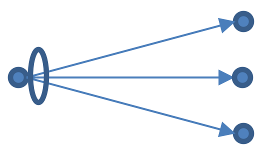
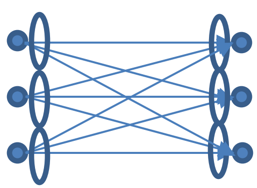
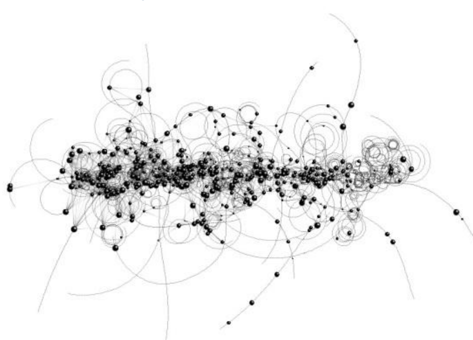
Information Theory of Networks

Topology	Capacity	Codes (Practical)
		
		
		
		
		

Information Theory of Networks

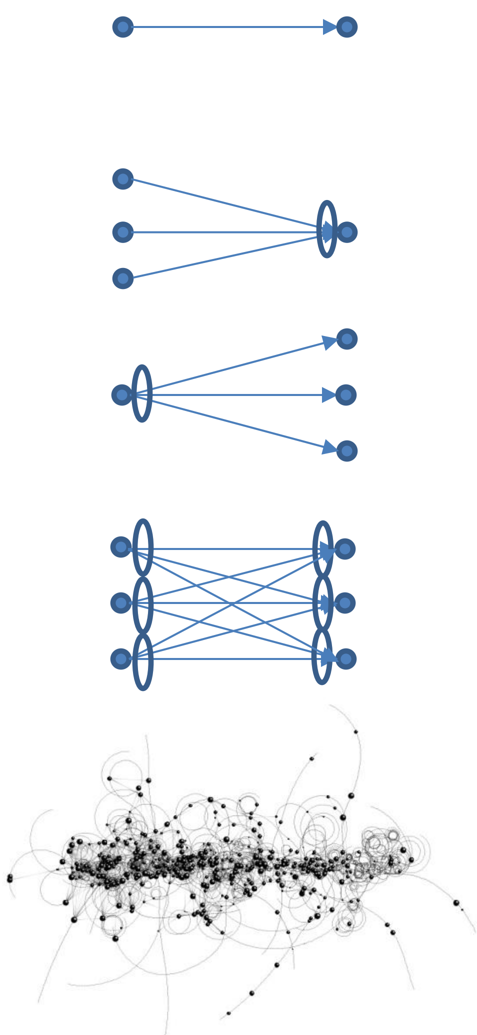
Topology	Capacity	Codes (Practical)
		Turbo Codes, Low-Density Parity-Check, Fountain Codes, Raptor Codes,
		
		
		
		

Information Theory of Networks

Topology	Capacity	Codes (Practical)
		Turbo Codes, Low-Density Parity-Check, Fountain Codes, Raptor Codes, Spinal Codes (Gaussian) Polar Codes (+ Multi-Access) Spatially-Coupled Codes (+MAC)
		
		
		
		

Information Theory of Networks

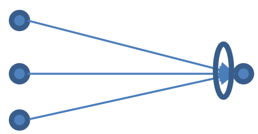
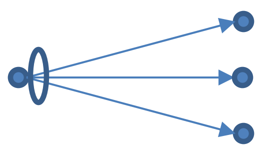
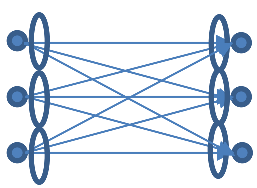
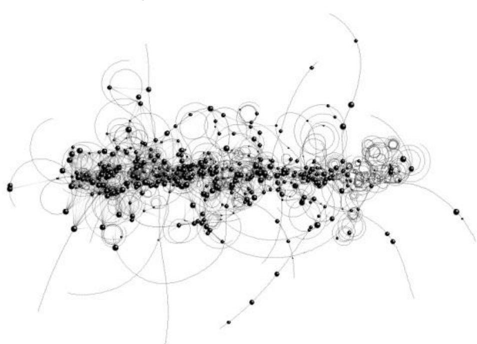
Topology	Capacity	Codes (Practical)



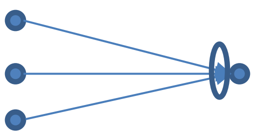
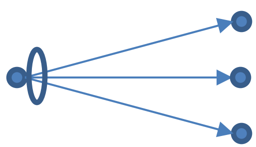
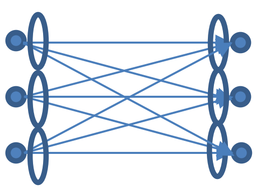
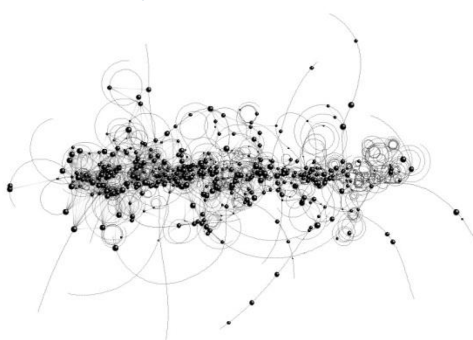
Turbo Codes,
Low-Density Parity-Check,
Fountain Codes, Raptor Codes,
Spinal Codes (Gaussian)
Polar Codes (+ Multi-Access)
Spatially-Coupled Codes (+MAC)

Broadcast Polar Codes

Information Theory of Networks

Topology	Capacity	Codes (Practical)
	✓	Turbo Codes, Low-Density Parity-Check, Fountain Codes, Raptor Codes, Spinal Codes (Gaussian) Polar Codes (+ Multi-Access) Spatially-Coupled Codes (+MAC)
	✓	
	✗ / ✓	**Broadcast Polar Codes**
	✗ / ✓	**Interference Alignment** **Computation Alignment**
		

Information Theory of Networks

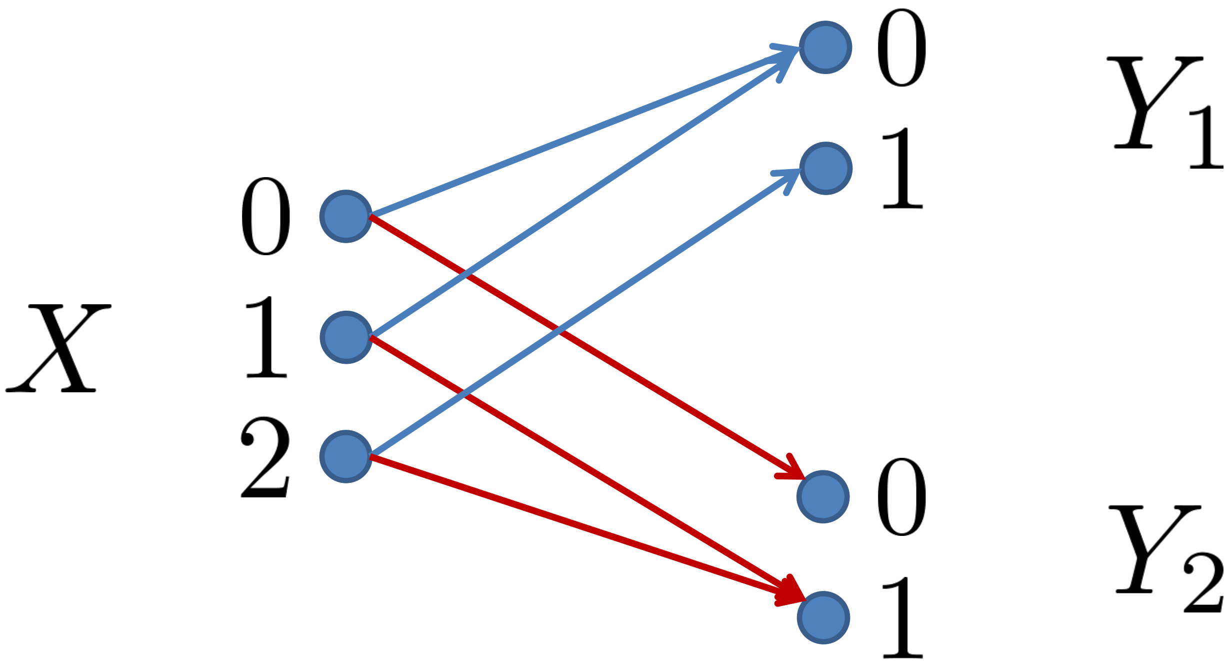
Topology	Capacity	Codes (Practical)
		Turbo Codes, Low-Density Parity-Check, Fountain Codes, Raptor Codes, Spinal Codes (Gaussian) Polar Codes (+ Multi-Access) Spatially-Coupled Codes (+MAC)
		
		**Broadcast Polar Codes**
		**Interference Alignment** **Computation Alignment**
		**Multicast Codes** **Multiple Unicast** **Un-Coded Transmission?**

Information Theory

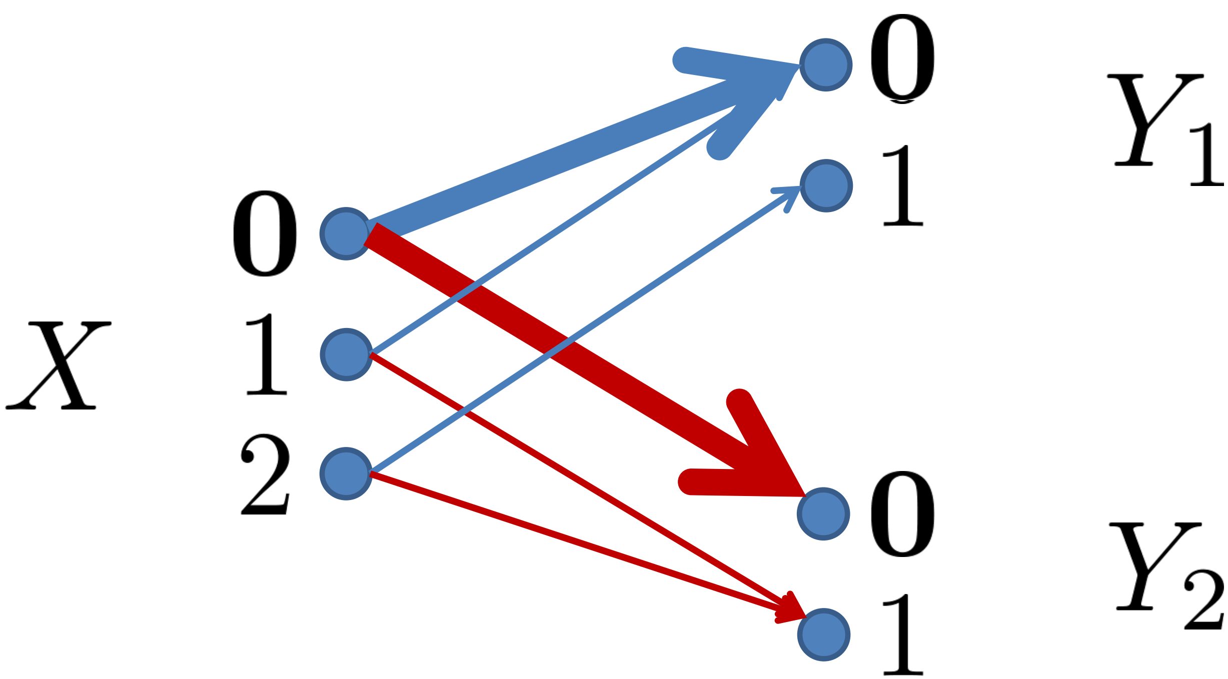
1. **Achievable Code**
2. **Converse Theorem**

Part I
A Simple Deterministic Broadcast Channel

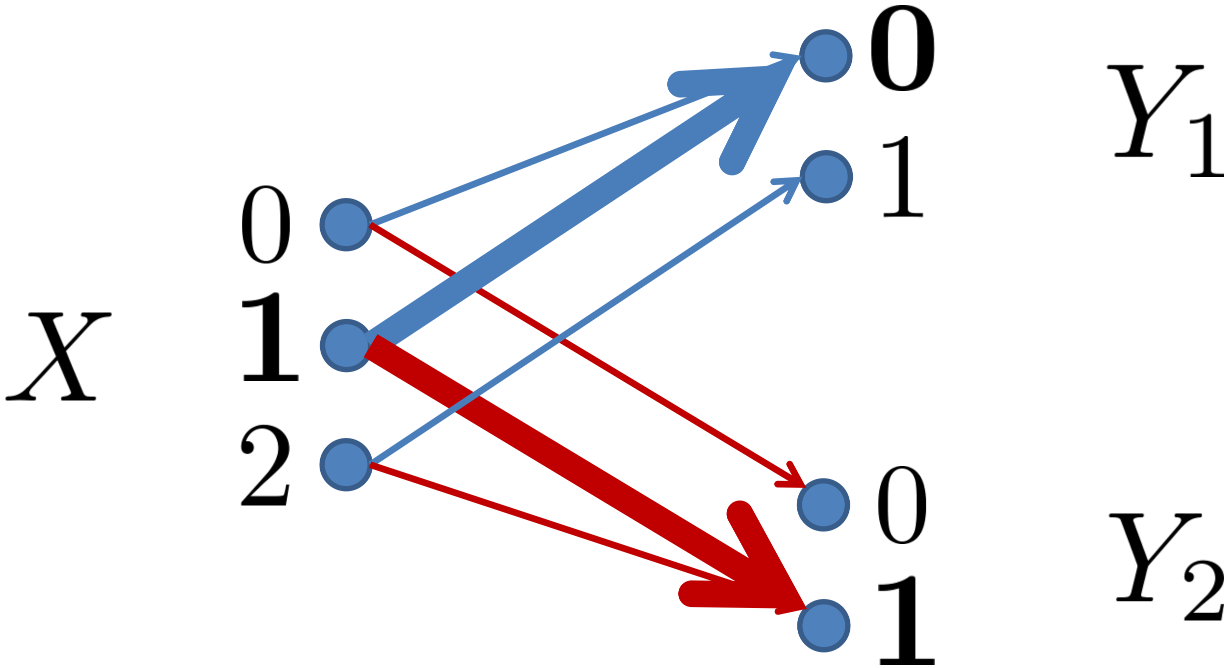
A Simple Deterministic Broadcast Channel



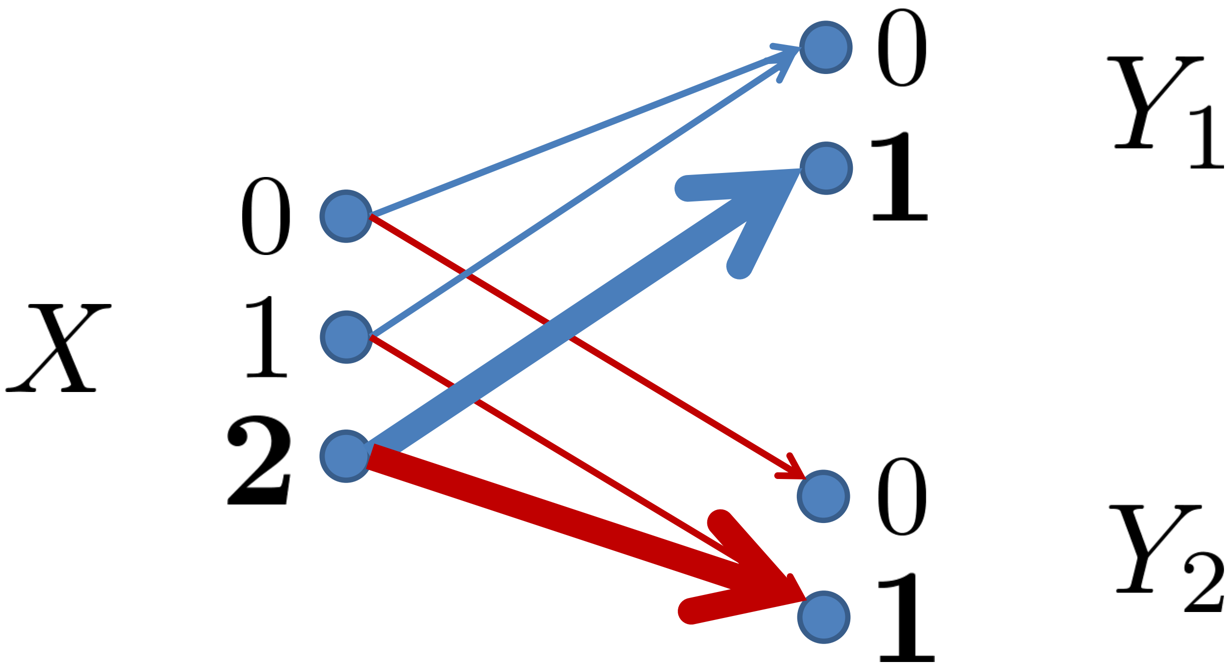
A Simple Deterministic Broadcast Channel



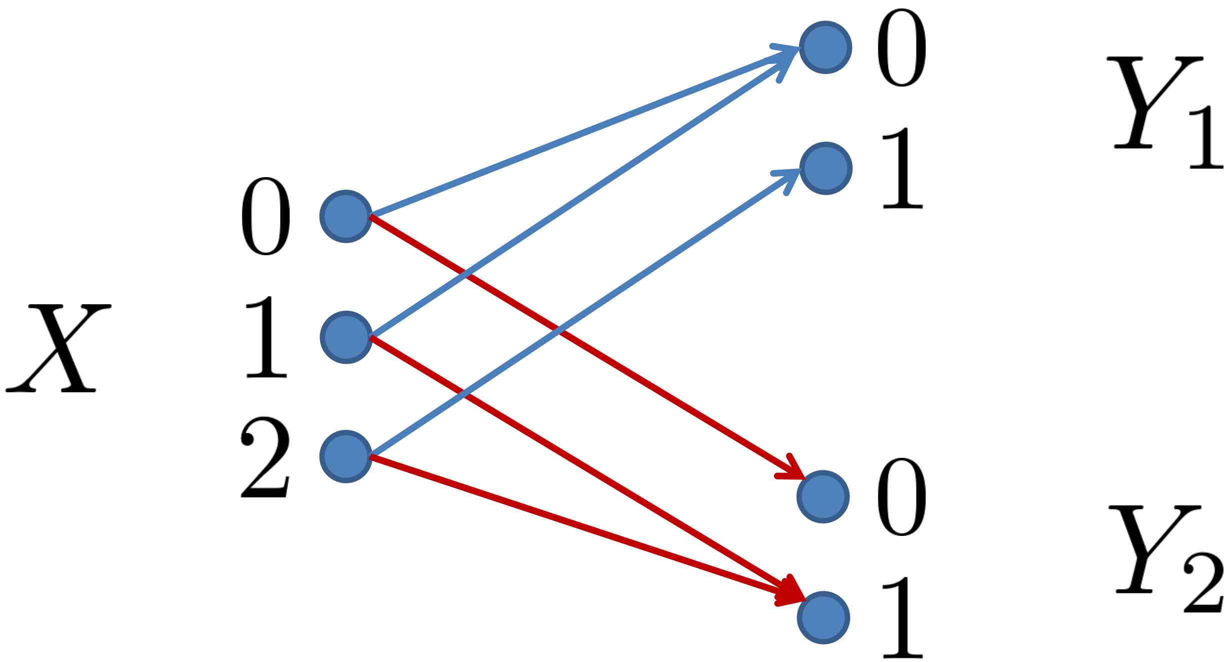
A Simple Deterministic Broadcast Channel



A Simple Deterministic Broadcast Channel



A Simple Deterministic Broadcast Channel

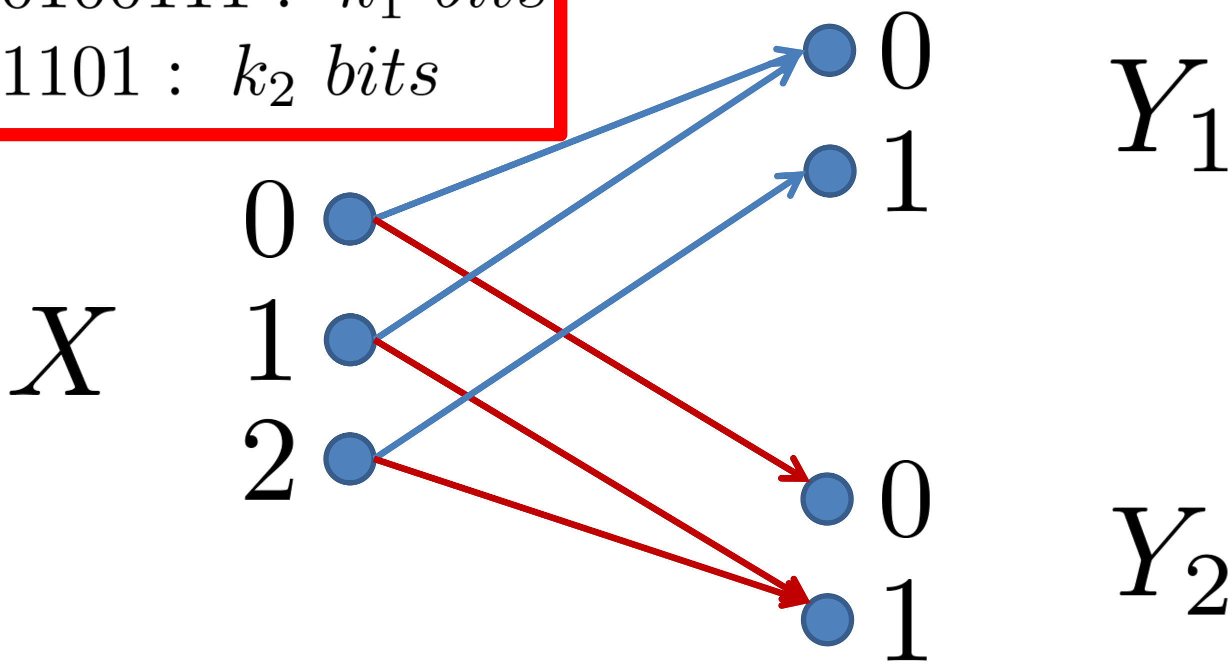


$$f_1(x) = \max(x-1, 0)$$
$$f_2(x) = \min(x, 1)$$
$$Y_i = f_i(X)$$

A Simple Deterministic Broadcast Channel

$W_1 : 0100111 : \; k_1 \; bits$

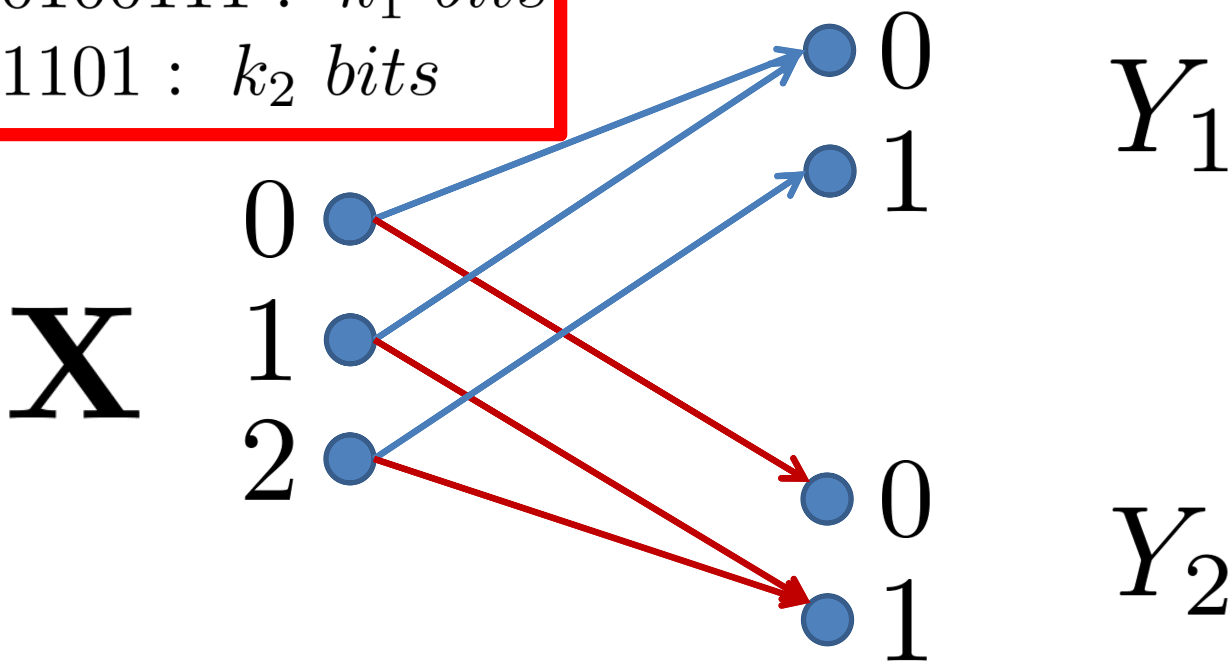
$W_2 : 1101 : \; k_2 \; bits$



A Simple Deterministic Broadcast Channel

$W_1 : 0100111 : \ k_1 \ bits$

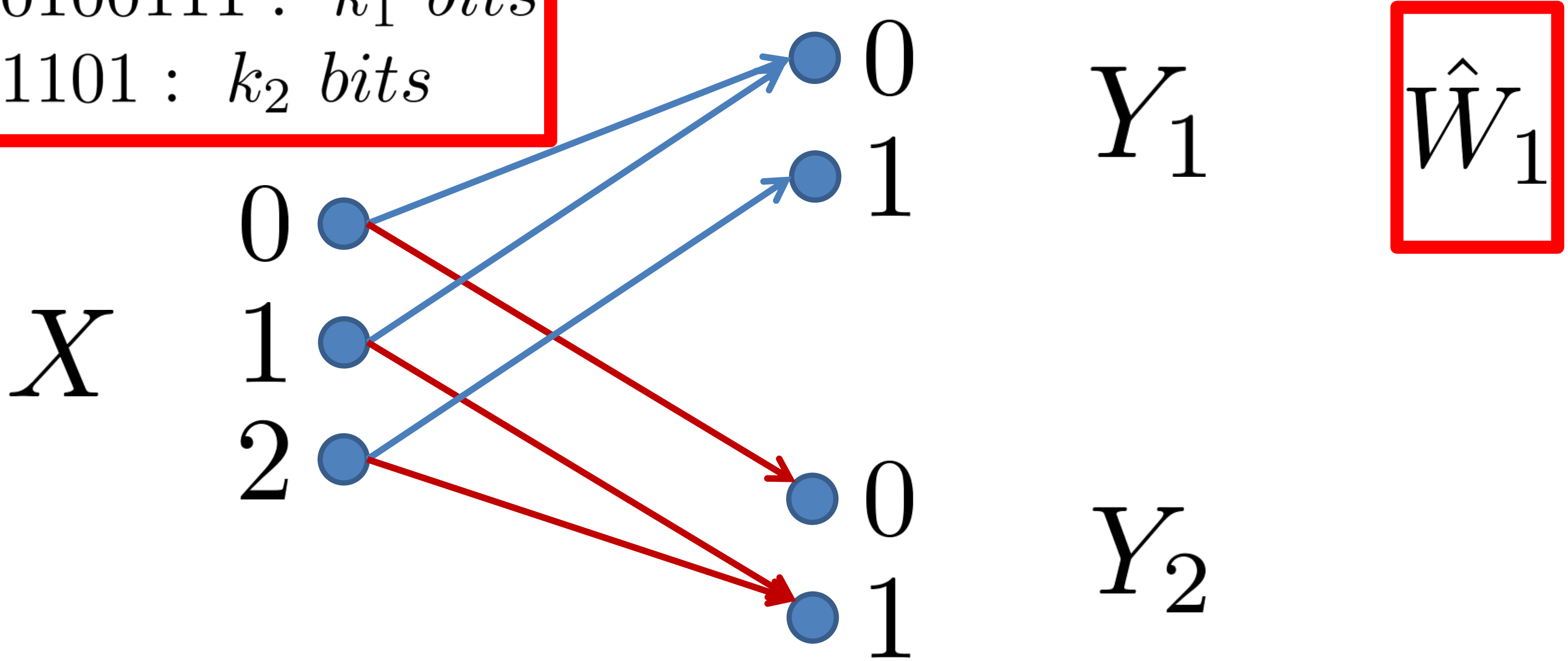
$W_2 : 1101 : \ k_2 \ bits$



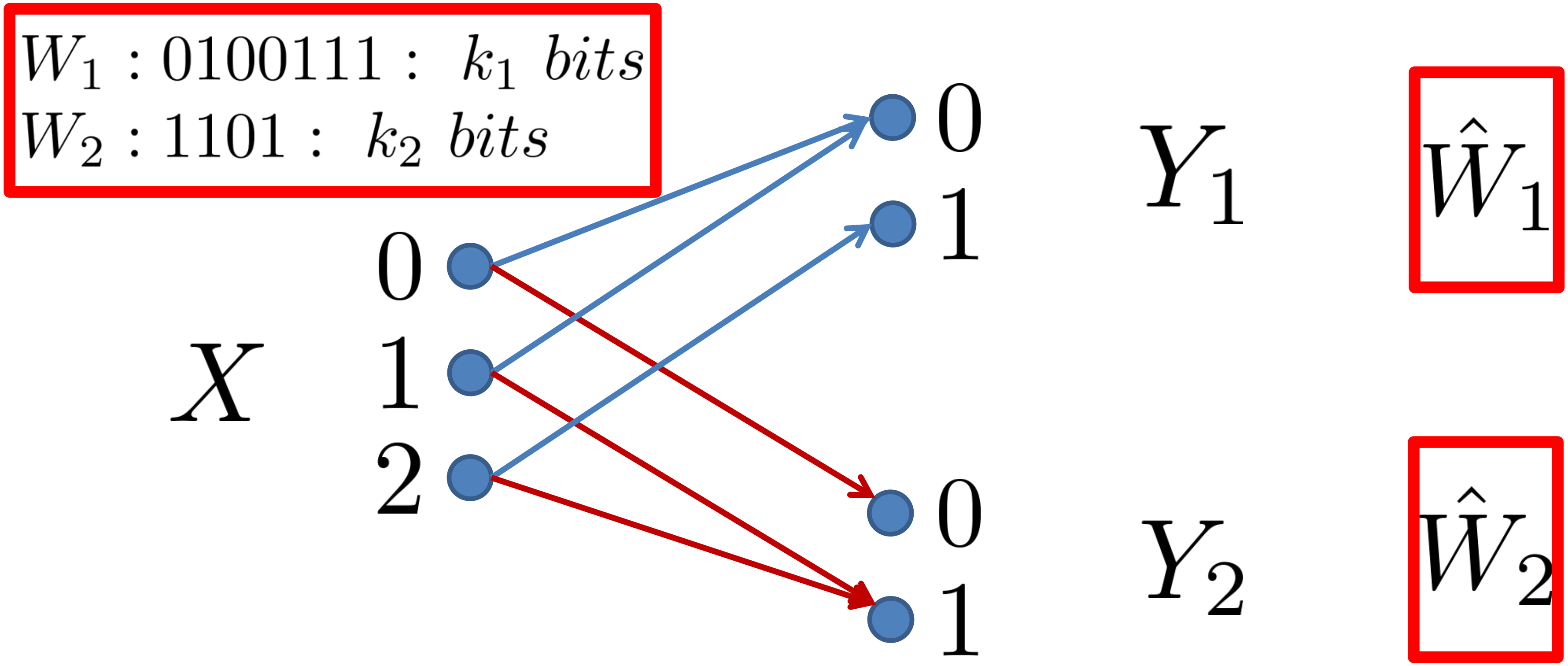
A Simple Deterministic Broadcast Channel

$W_1 : 0100111 : \; k_1 \; bits$

$W_2 : 1101 : \; k_2 \; bits$



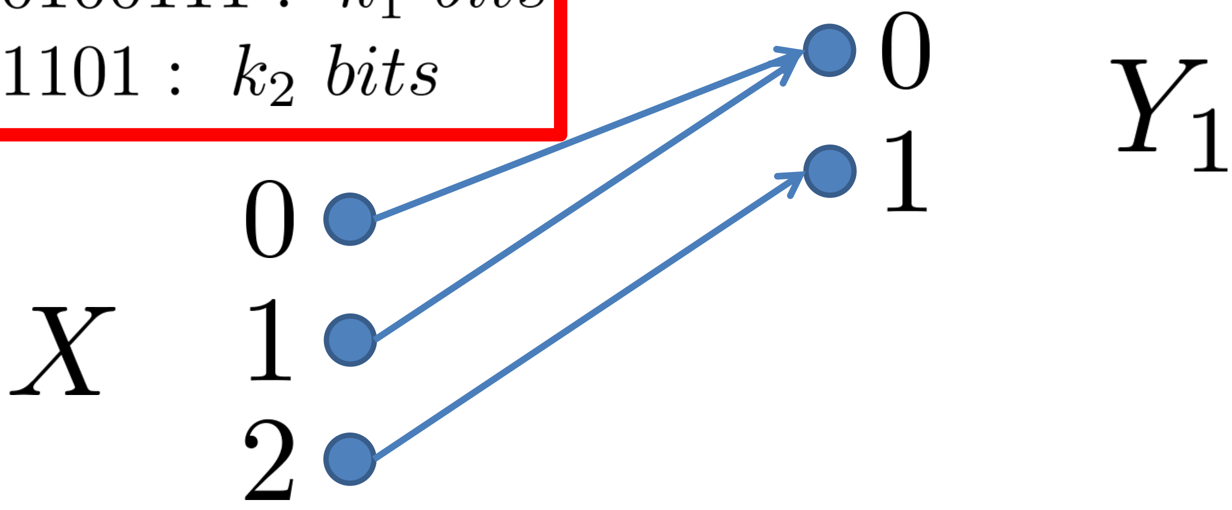
A Simple Deterministic Broadcast Channel



A Simple Deterministic Broadcast Channel

$W_1 : 0100111 : \; k_1 \; bits$

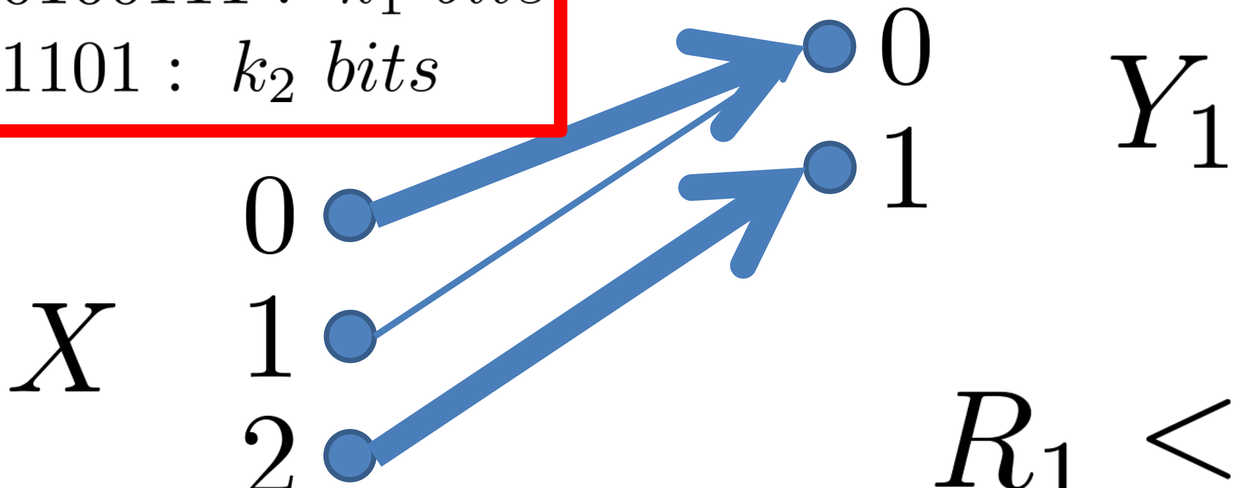
$W_2 : 1101 : \; k_2 \; bits$



A Simple Deterministic Broadcast Channel

$W_1 : 0100111 : \ k_1 \ bits$

$W_2 : 1101 : \ k_2 \ bits$



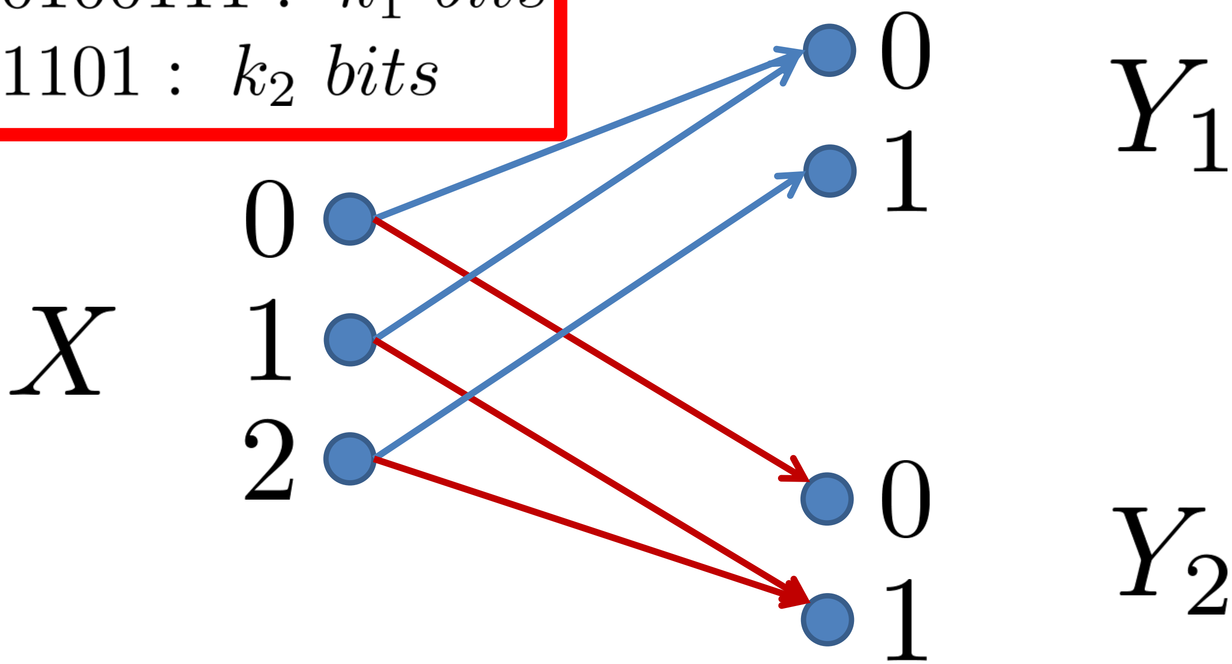
$$R_1 \leq 1$$

(Ignoring 2nd Receiver)

A Simple Deterministic Broadcast Channel

$W_1 : 0100111 : \ k_1 \ bits$

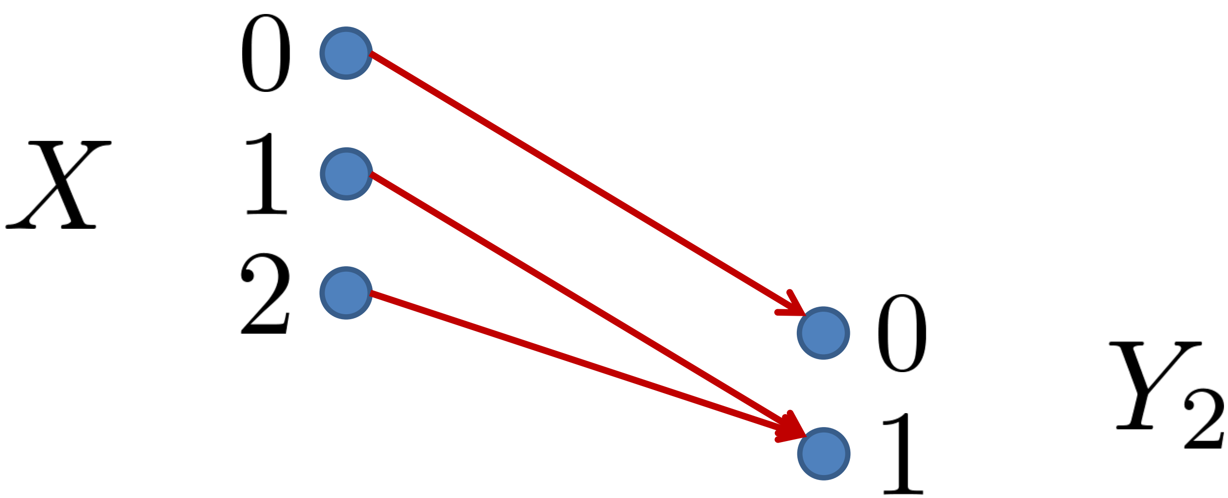
$W_2 : 1101 : \ k_2 \ bits$



A Simple Deterministic Broadcast Channel

$W_1 : 0100111 : \ k_1 \ bits$

$W_2 : 1101 : \ k_2 \ bits$

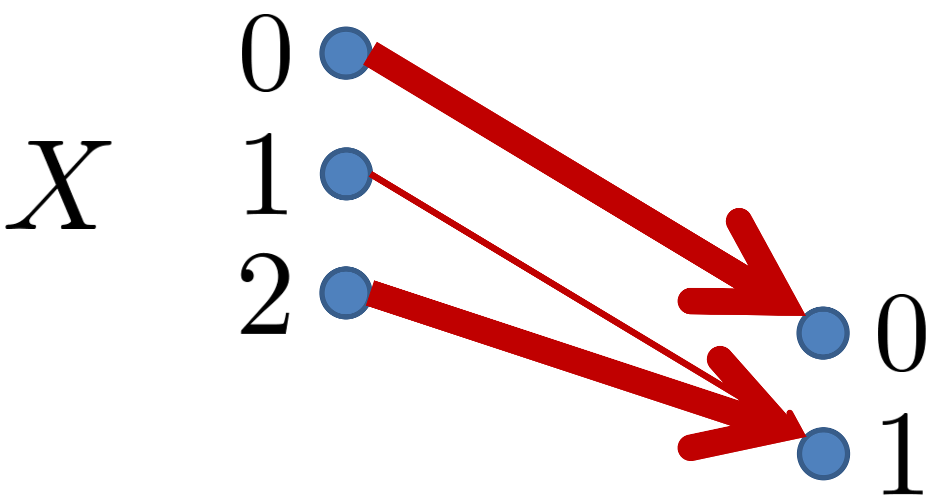


A Simple Deterministic Broadcast Channel

$W_1 : 0100111 : \ k_1 \ bits$
$W_2 : 1101 : \ k_2 \ bits$

(Ignoring 1st Receiver)

$$R_2 \leq 1$$

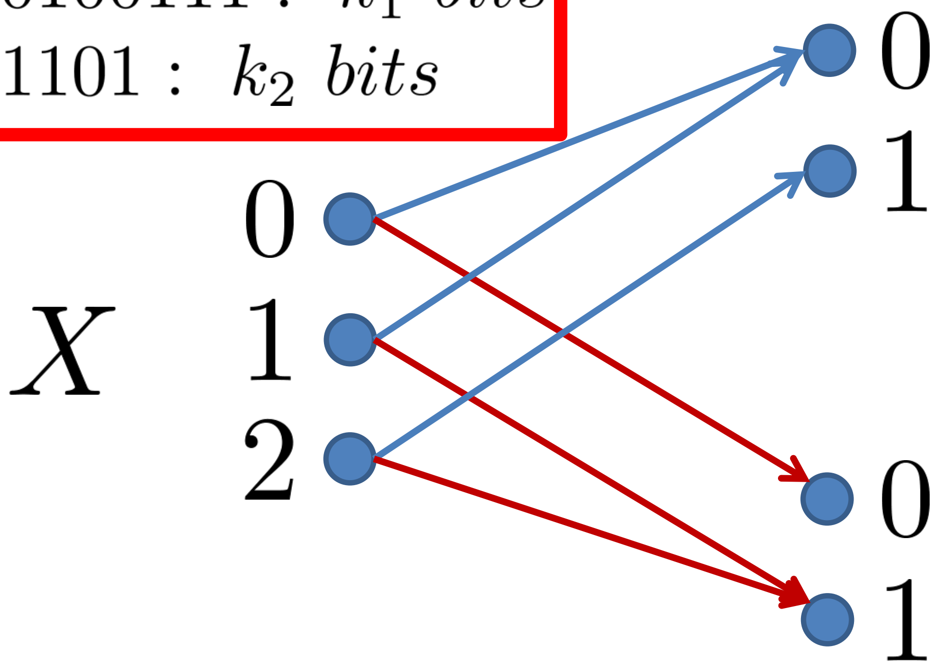


Y_2

A Simple Deterministic Broadcast Channel

$W_1 : 0100111 : \ k_1 \ bits$

$W_2 : 1101 : \ k_2 \ bits$



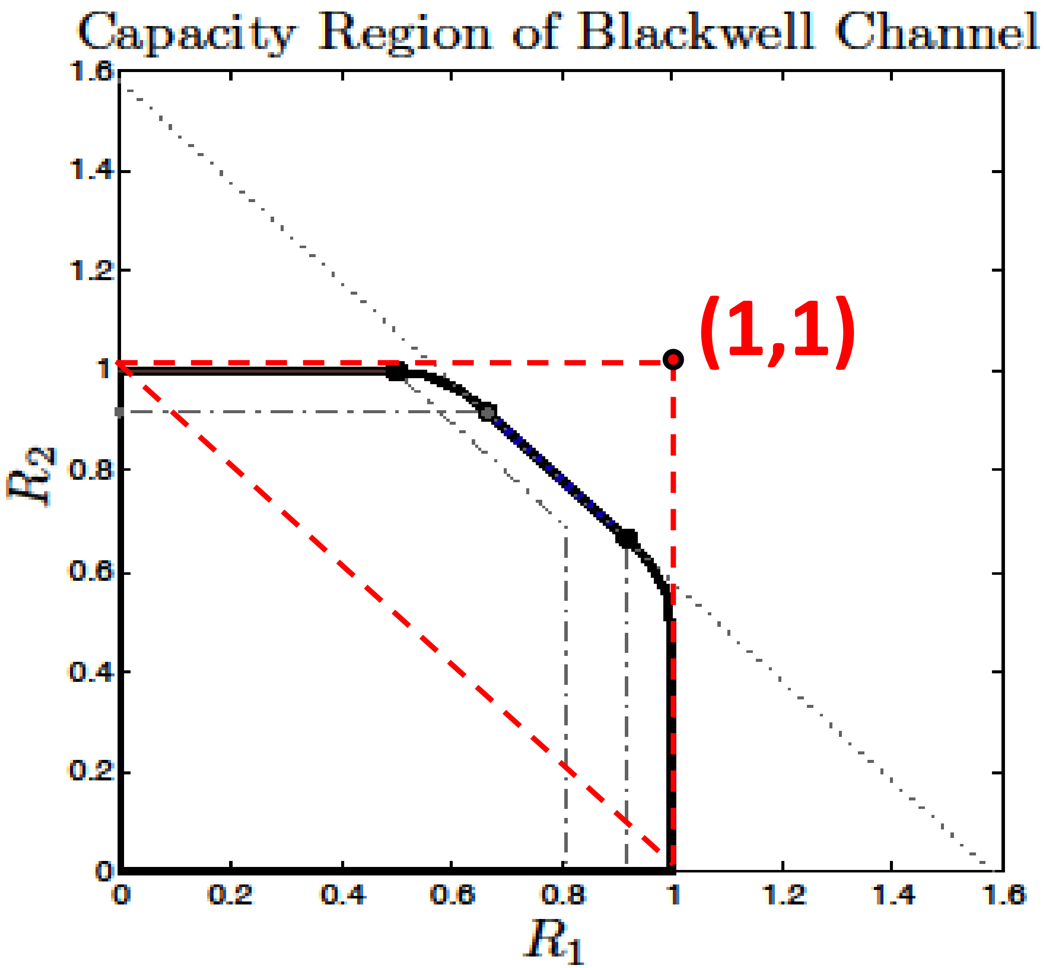
Y_1

Capacity?

Y_2

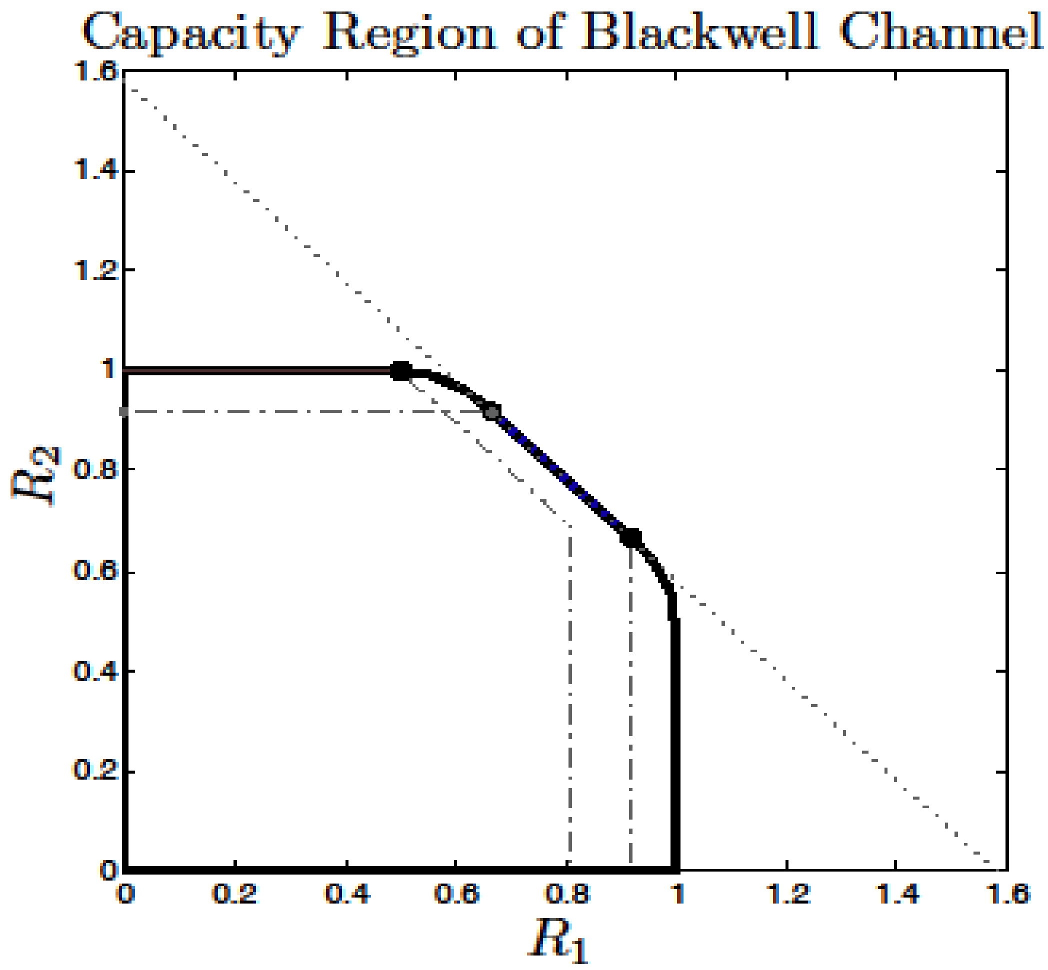
$$R_1 = \frac{k_1}{n} \qquad R_2 = \frac{k_2}{n}$$

A Simple Deterministic Broadcast Channel

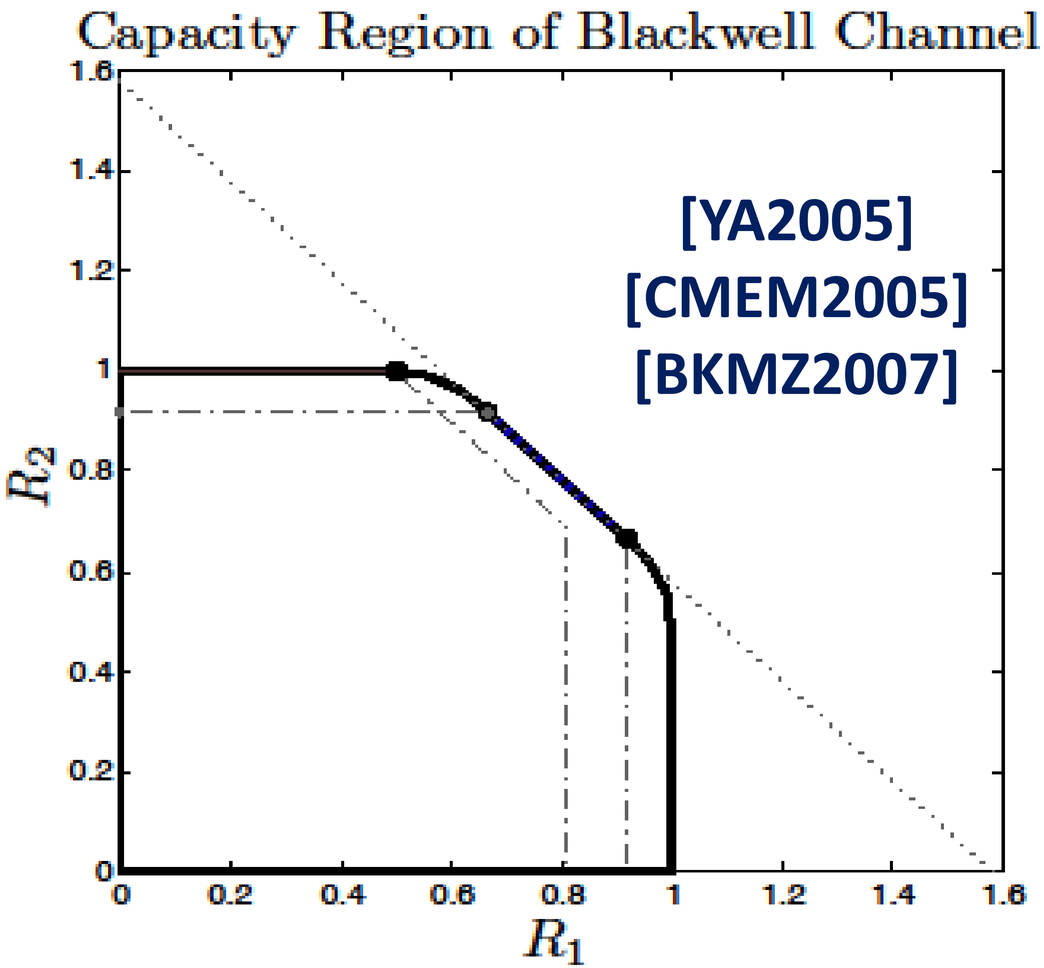


[Marton77][Pinsker78]

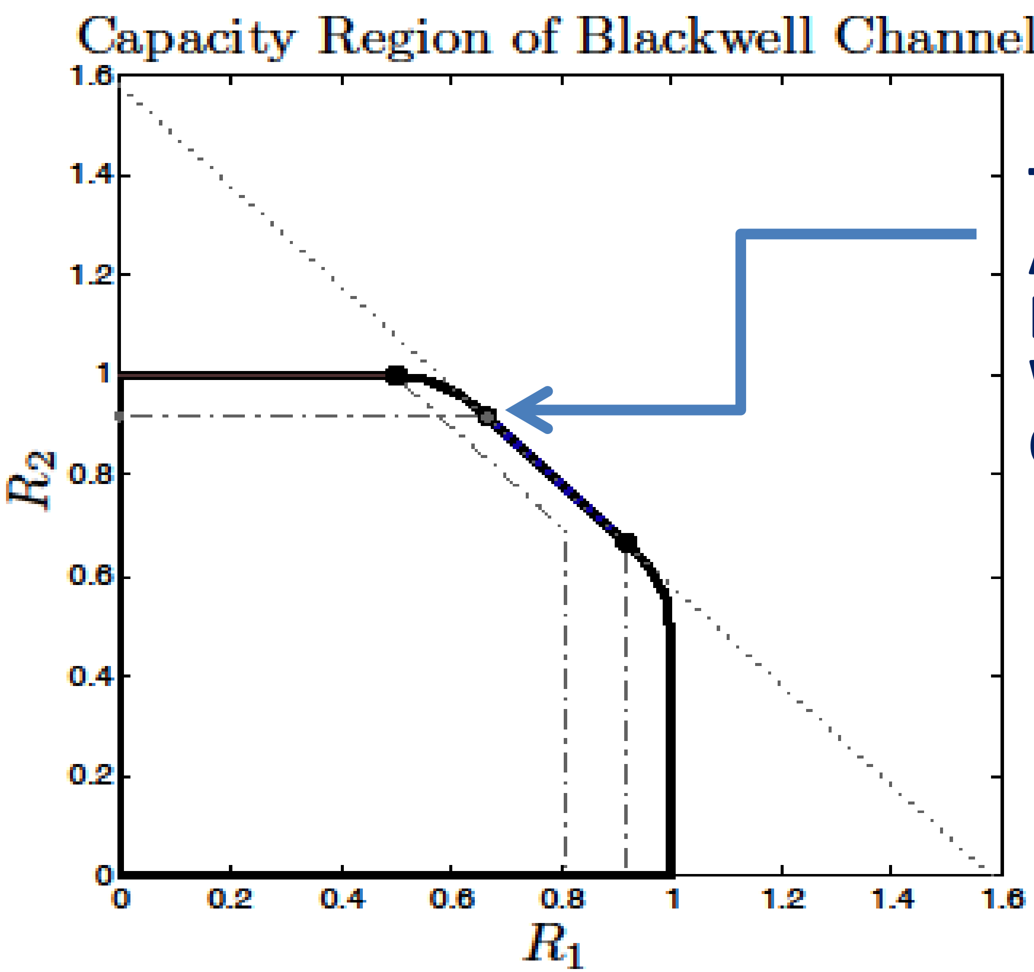
A Simple Deterministic Broadcast Channel



A Simple Deterministic Broadcast Channel



A Simple Deterministic Broadcast Channel



Theorem: Can Achieve Capacity Boundary Points With Low-Complexity

A Simple Deterministic Broadcast Channel

	Shannon	N.G., Abbe, Gastpar
Complexity	2^n	$n \log n$
$P_e^{(n)}$	2^{-n}	$2^{-\sqrt{n}}$

A Simple Deterministic Broadcast Channel

	Shannon	N.G., Abbe, Gastpar
Complexity	2^n	$n \log n$
$P_e^{(n)}$	2^{-n}	$2^{-\sqrt{n}}$

A Simple Deterministic Broadcast Channel

	Shannon	N.G., Abbe, Gastpar
Complexity	2^n	$n \log n$
$P_e^{(n)}$	2^{-n}	$2^{-\sqrt{n}}$

A Simple Deterministic Broadcast Channel

	Shannon	N.G., Abbe, Gastpar
Complexity	2^n	$n \log n$
$P_e^{(n)}$	2^{-n}	$2^{-\sqrt{n}}$

Point-to-Point Channel (DMC)

$$P_e^{(n)} = n^{-\frac{1}{4}}$$

[Arikan2008]

A Simple Deterministic Broadcast Channel

	Shannon	N.G., Abbe, Gastpar
Complexity	2^n	$n \log n$
$P_e^{(n)}$	2^{-n}	$2^{-\sqrt{n}}$

Point-to-Point Channel (DMC)

$$P_e^{(n)} = 2^{-\sqrt{n}}$$

[ArikanTelatar2009]

A Simple Deterministic Broadcast Channel

	Shannon	N.G., Abbe, Gastpar
Complexity	2^n	$n \log n$
$P_e^{(n)}$	2^{-n}	$2^{-\sqrt{n}}$

Point-to-Point Channel (DMC)

$$P_e^{(n)} = 2^{-2^{\frac{\ell}{2}+\sqrt{\ell}Q^{-1}\left(\frac{R}{I(W)}\right)+o(\ell)}}$$

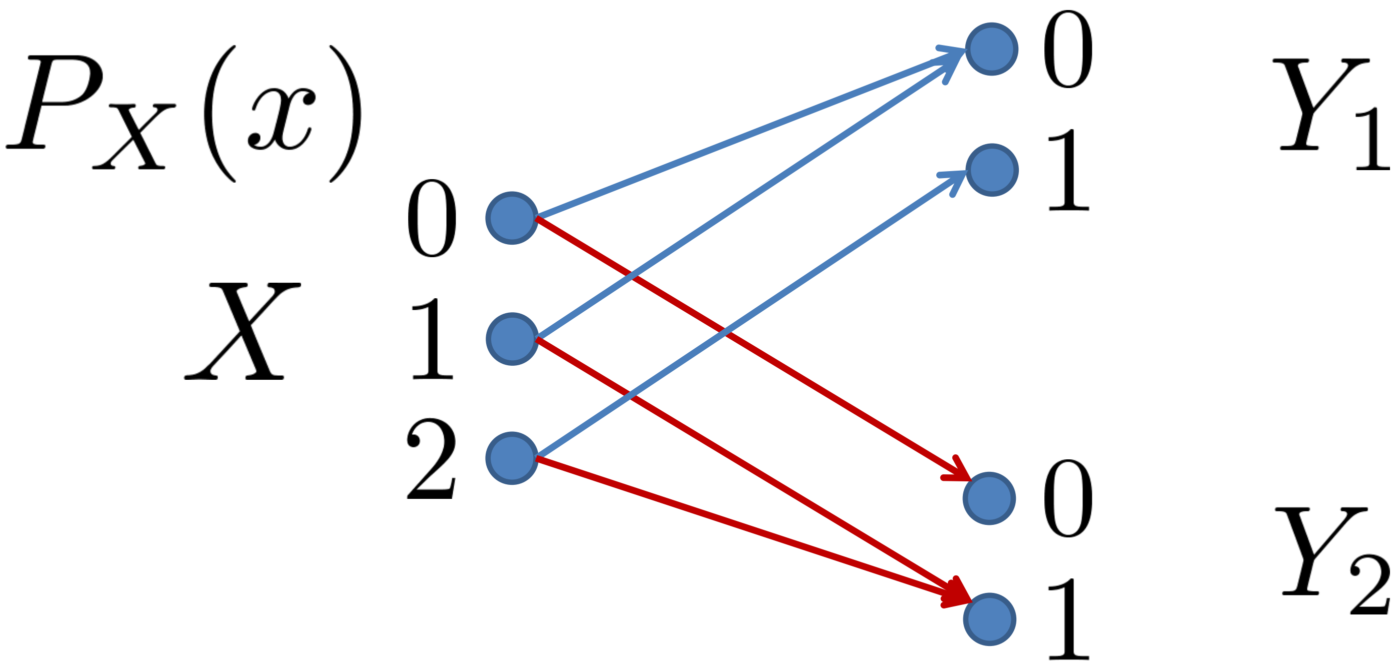
[HMTU2013]

$\ell \equiv \log_2 n$

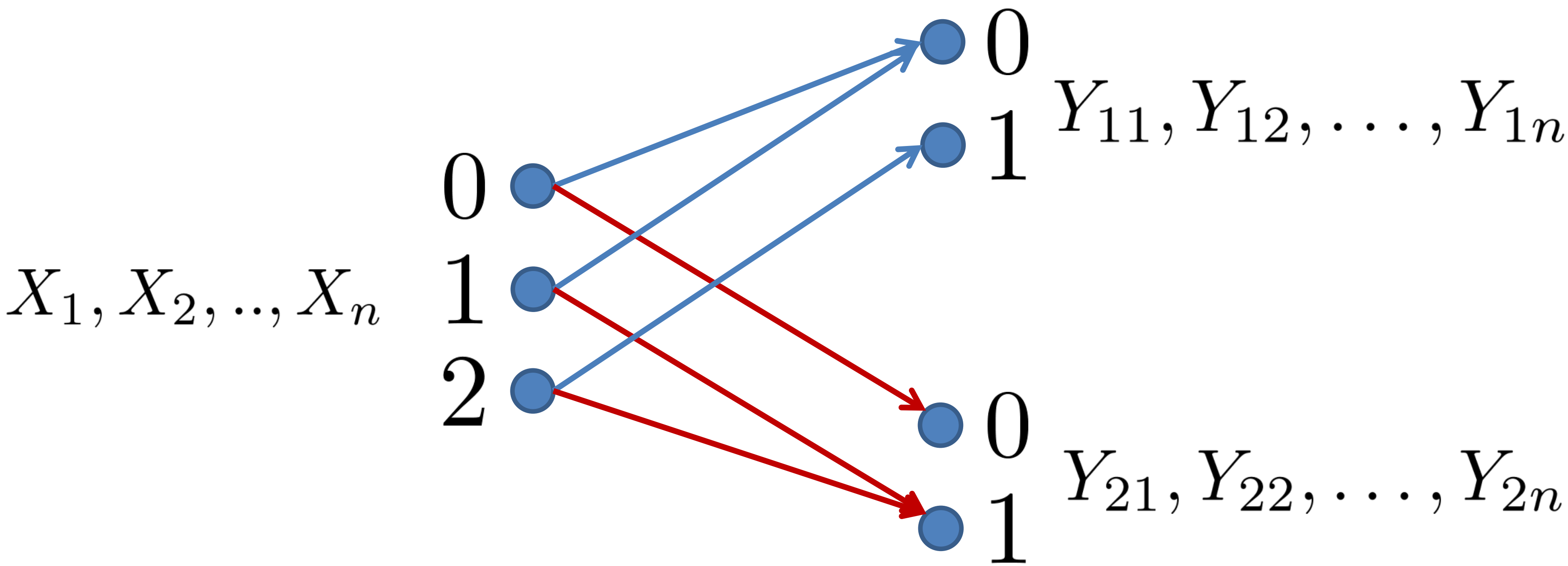
A Simple Deterministic Broadcast Channel

	Shannon	N.G., Abbe, Gastpar
Complexity	2^n	$n \log n$
$P_e^{(n)}$	2^{-n}	$2^{-\sqrt{n}}$

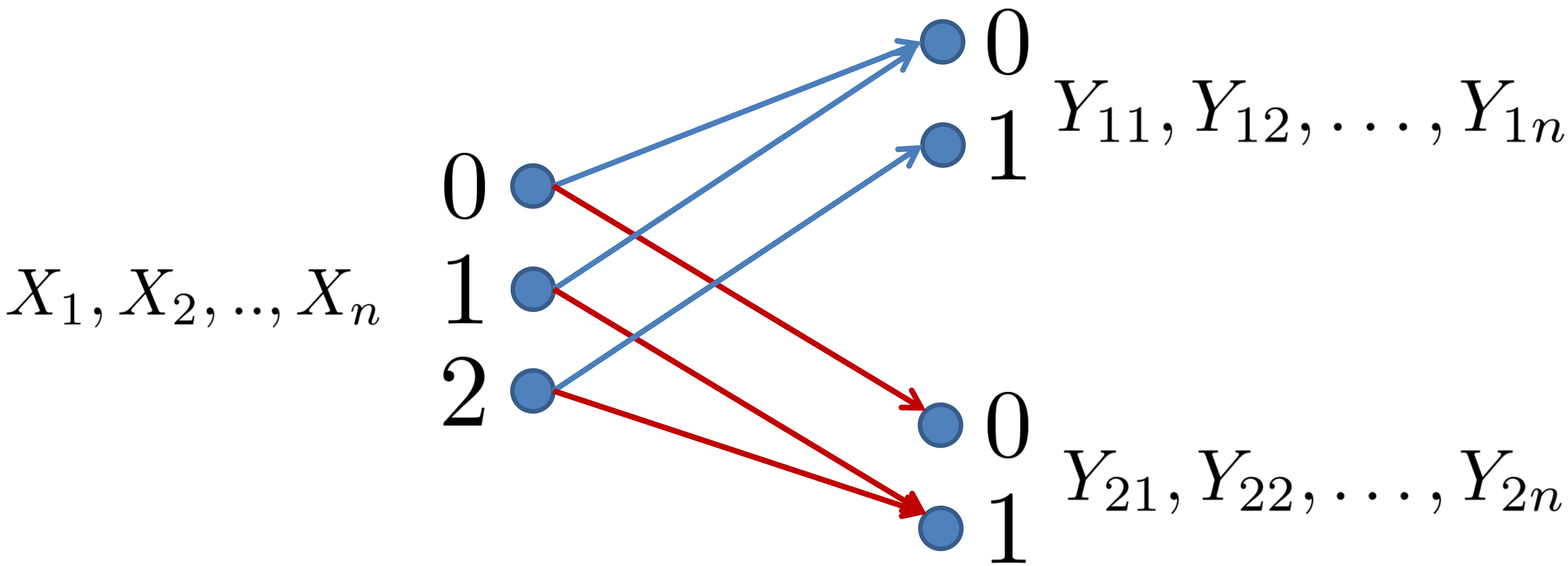
Sketch: Polar Code For Deterministic Broadcast



Sketch: Polar Code For Deterministic Broadcast

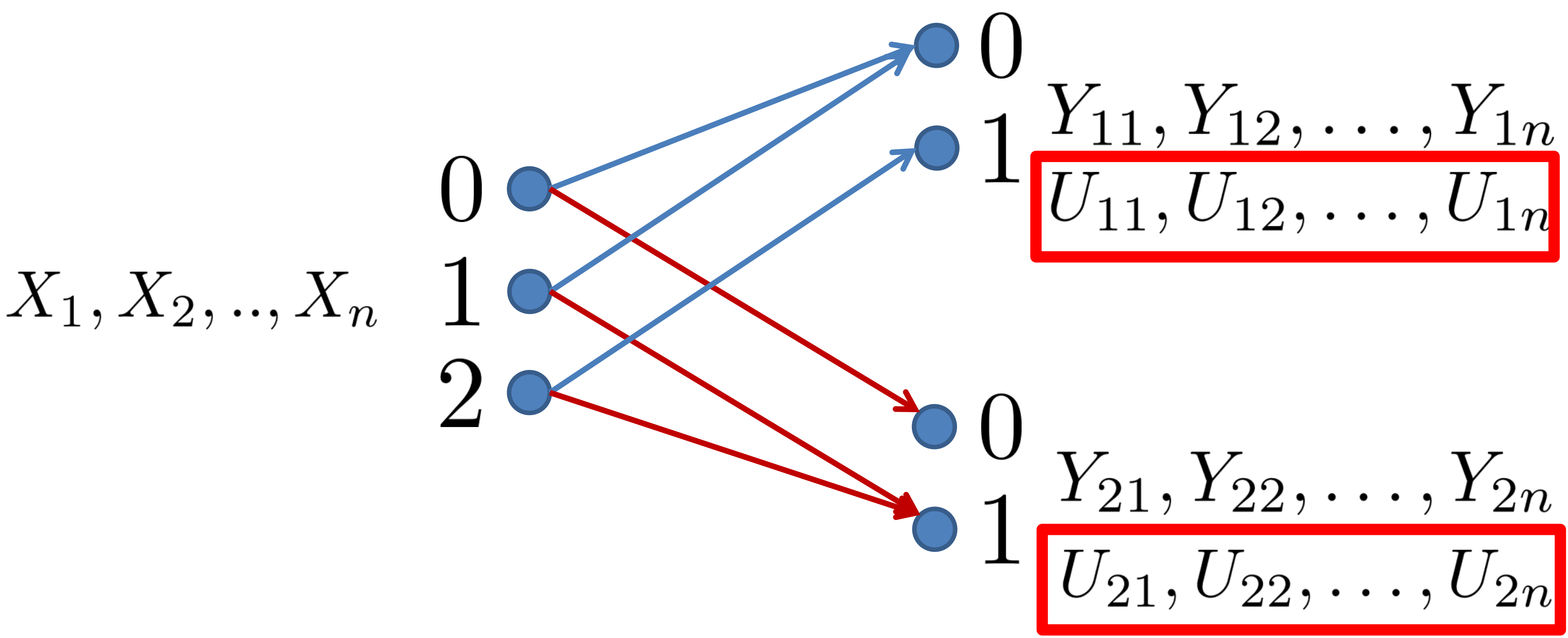


Sketch: Polar Code For Deterministic Broadcast



Polarize Output Random Variables

Sketch: Polar Code For Deterministic Broadcast



Polarize Output Random Variables

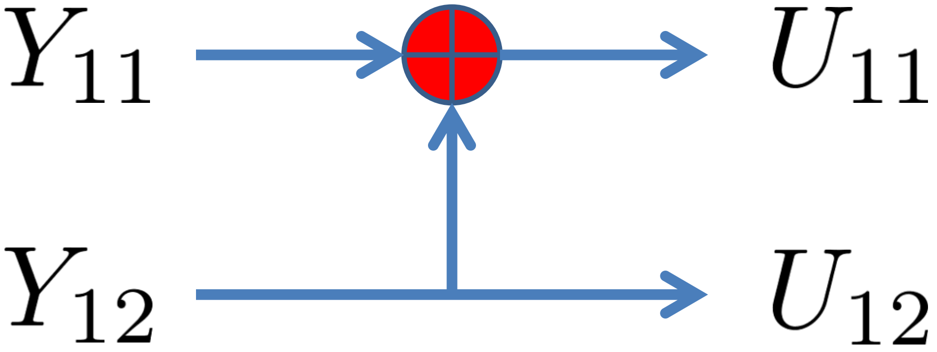
Sketch: Polar Code For Deterministic Broadcast

Y_{11}

Y_{12}

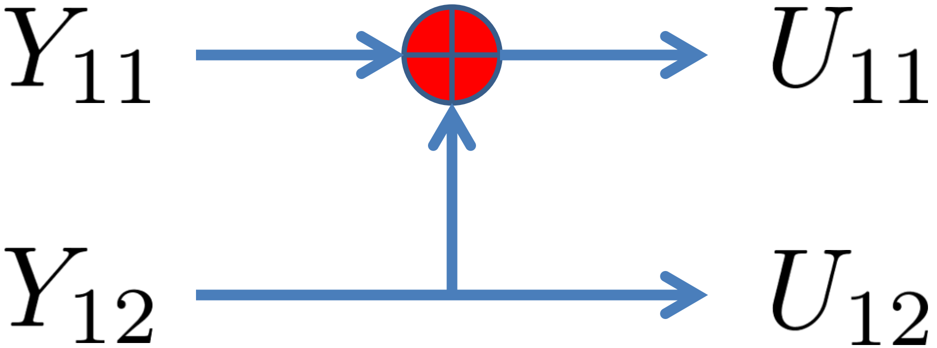
Sketch: Polar Code For Deterministic Broadcast

Create Dependencies



Sketch: Polar Code For Deterministic Broadcast

Create Dependencies

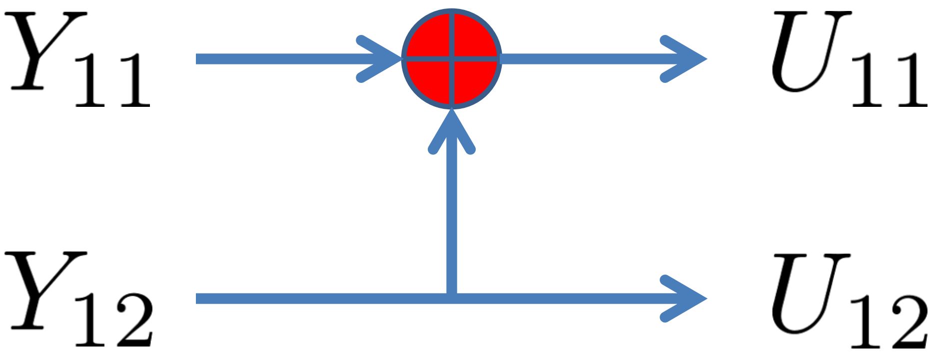


$$H(U_{11}) \geq H(Y_{11})$$

$$H(U_{12}|U_{11}) \leq H(Y_{12})$$

Sketch: Polar Code For Deterministic Broadcast

Create Dependencies



$$H(U_{11}) \geq H(Y_{11})$$

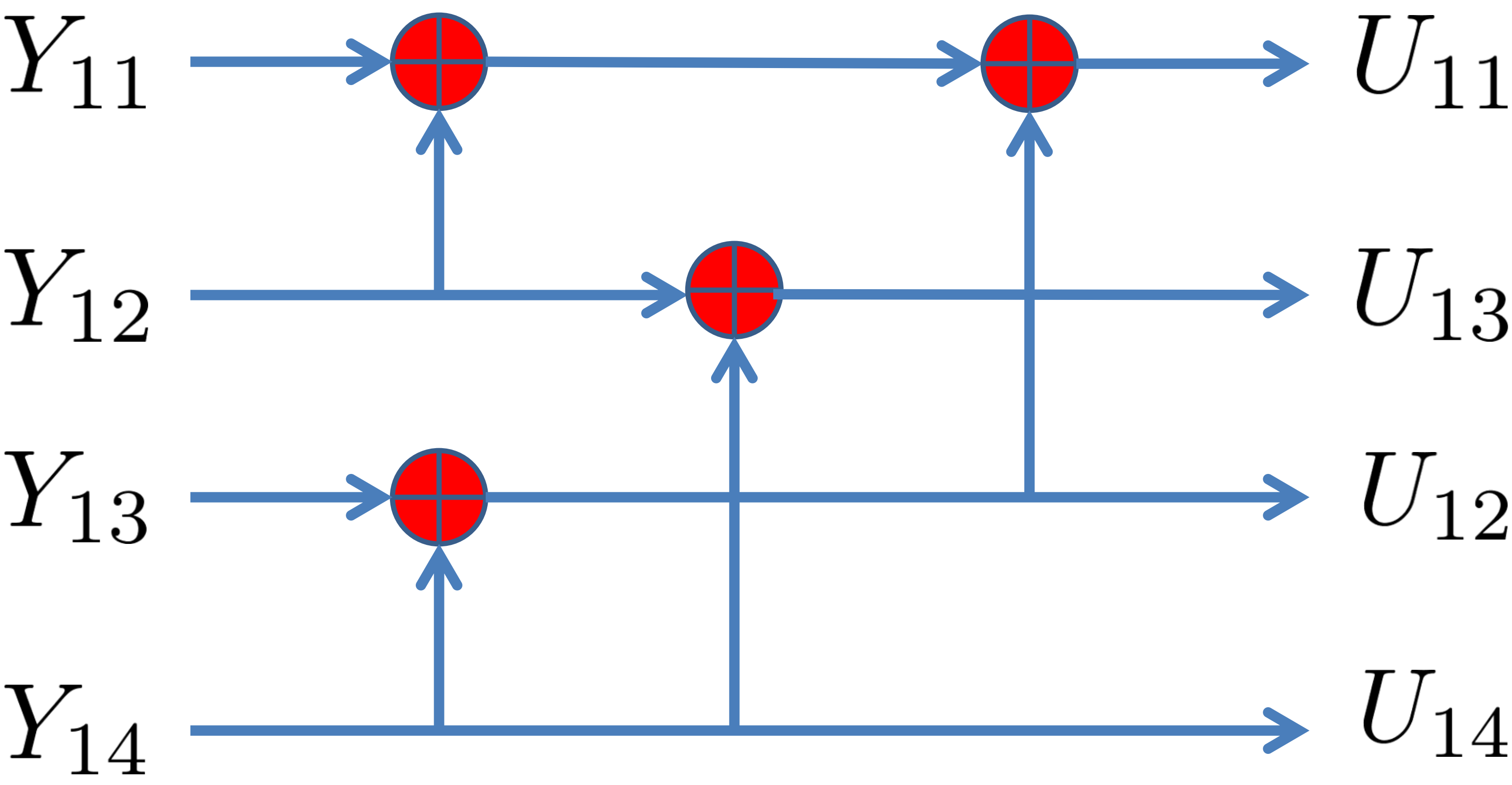
$$H(U_{12}|U_{11}) \leq H(Y_{12})$$

$$H(U_{11}) + H(U_{12}|U_{11}) = H(U_{11}U_{12}) = H(Y_{11}) + H(Y_{12})$$

Conservation of Entropy

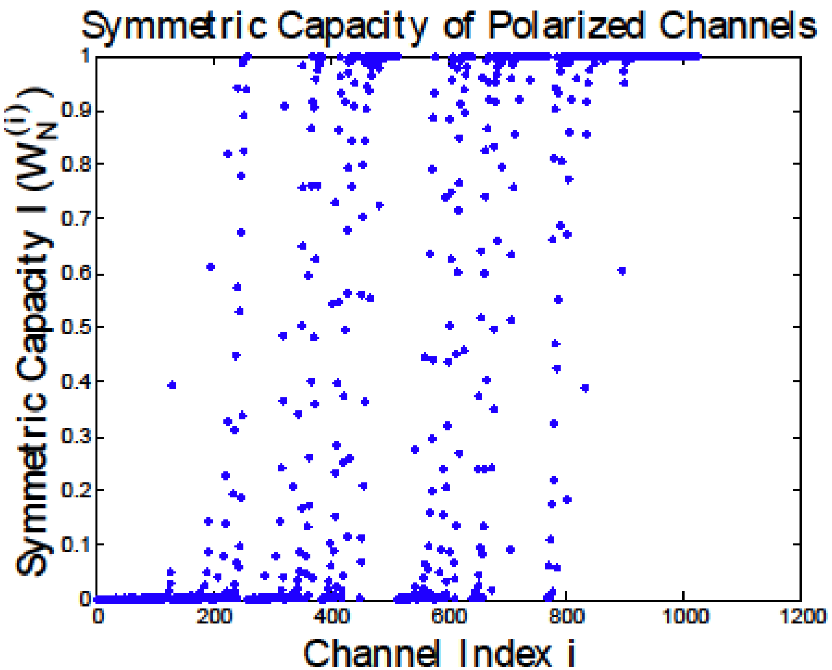
Sketch: Polar Code For Deterministic Broadcast

Create Dependencies

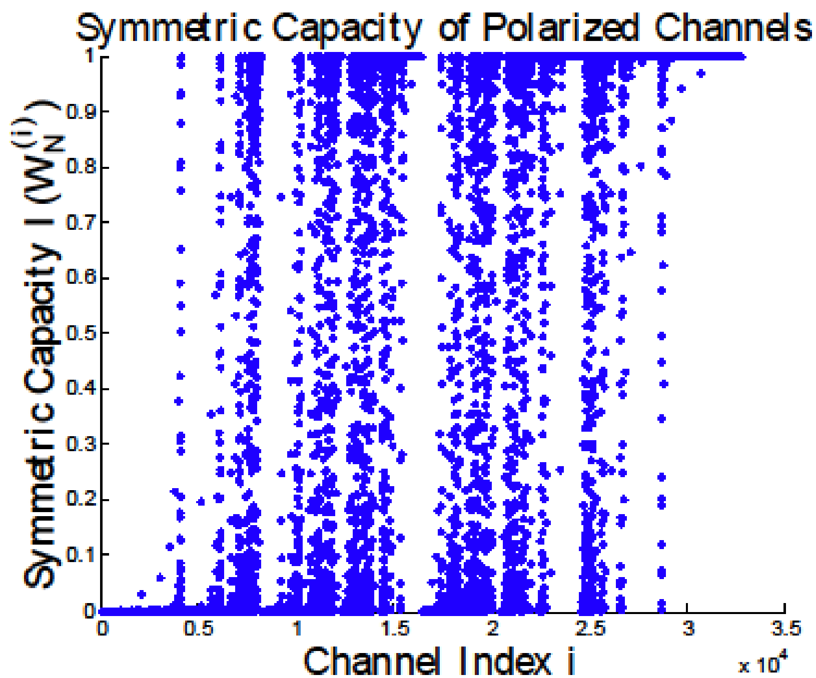


Polarization Theory

Aside… polarization theory for channels, sources, random variables

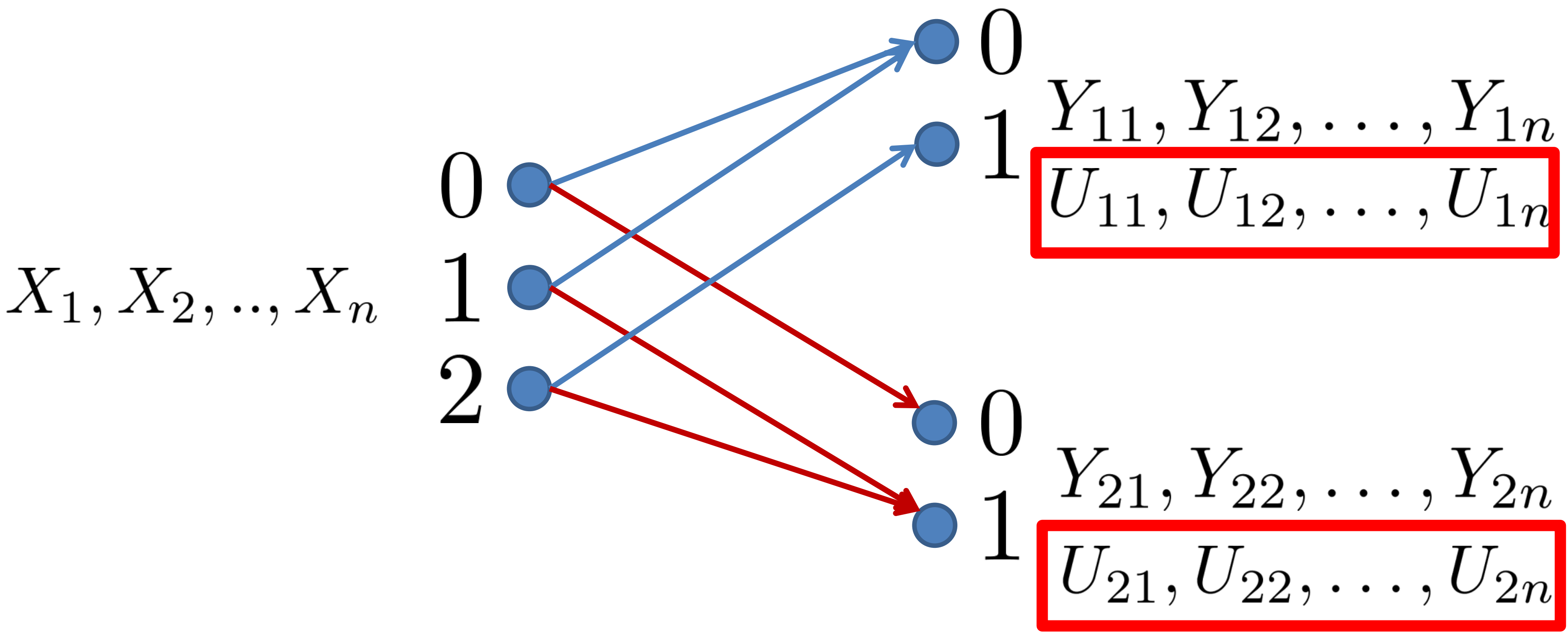


$\mathbf{N} = 2^{10}$



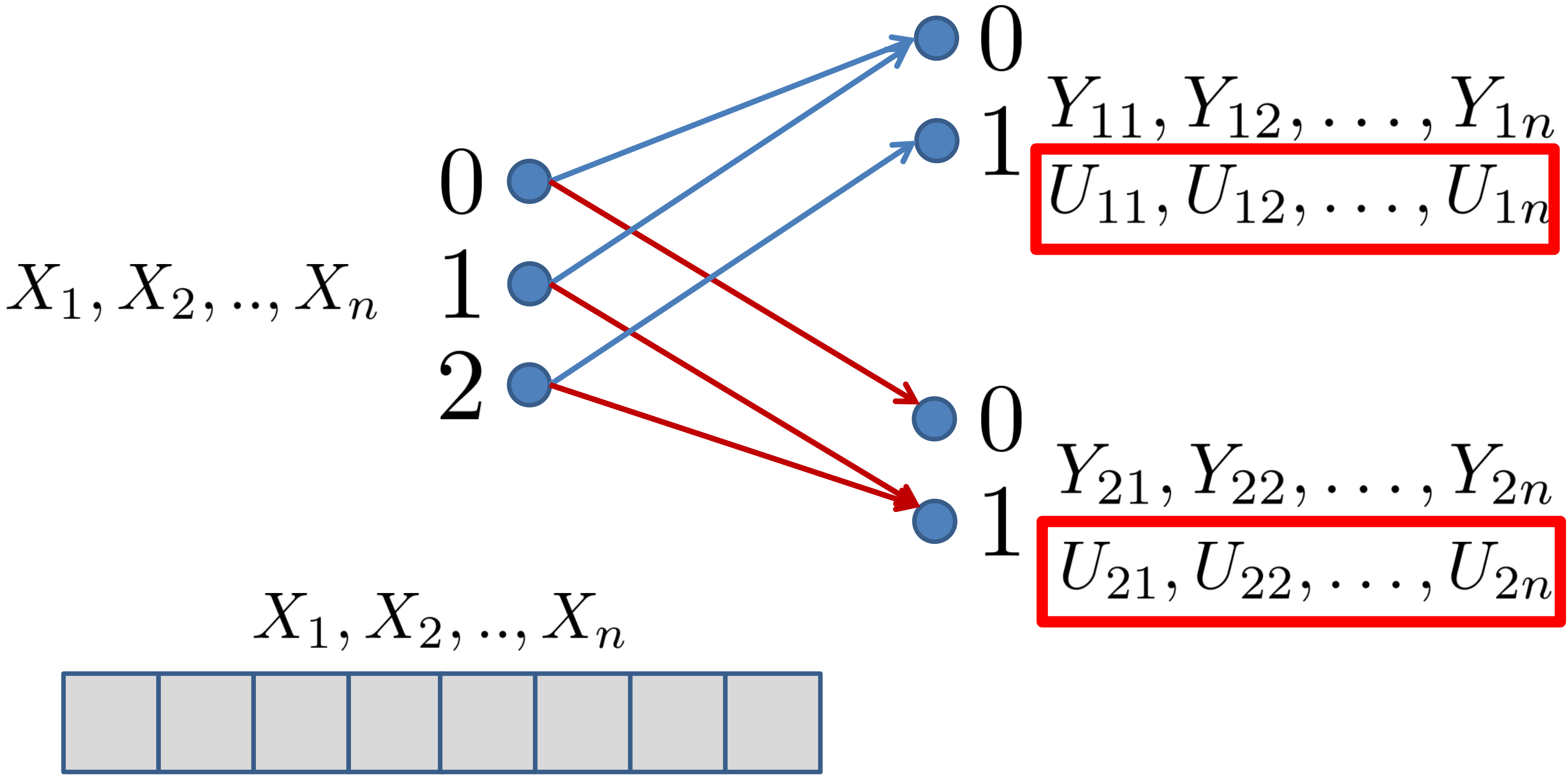
$\mathbf{N} = 2^{15}$

Sketch: Polar Code For Deterministic Broadcast

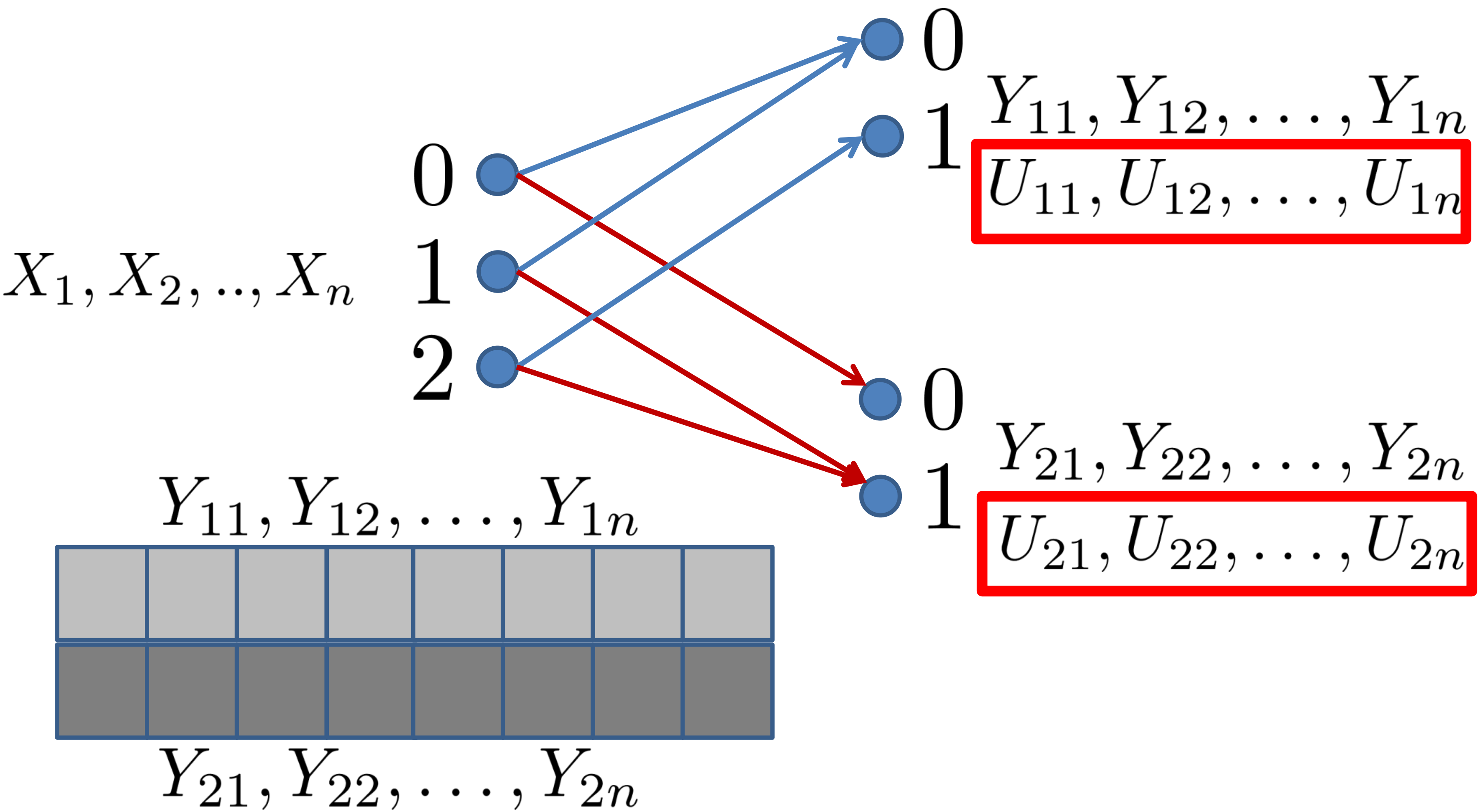


Polarize Output Random Variables

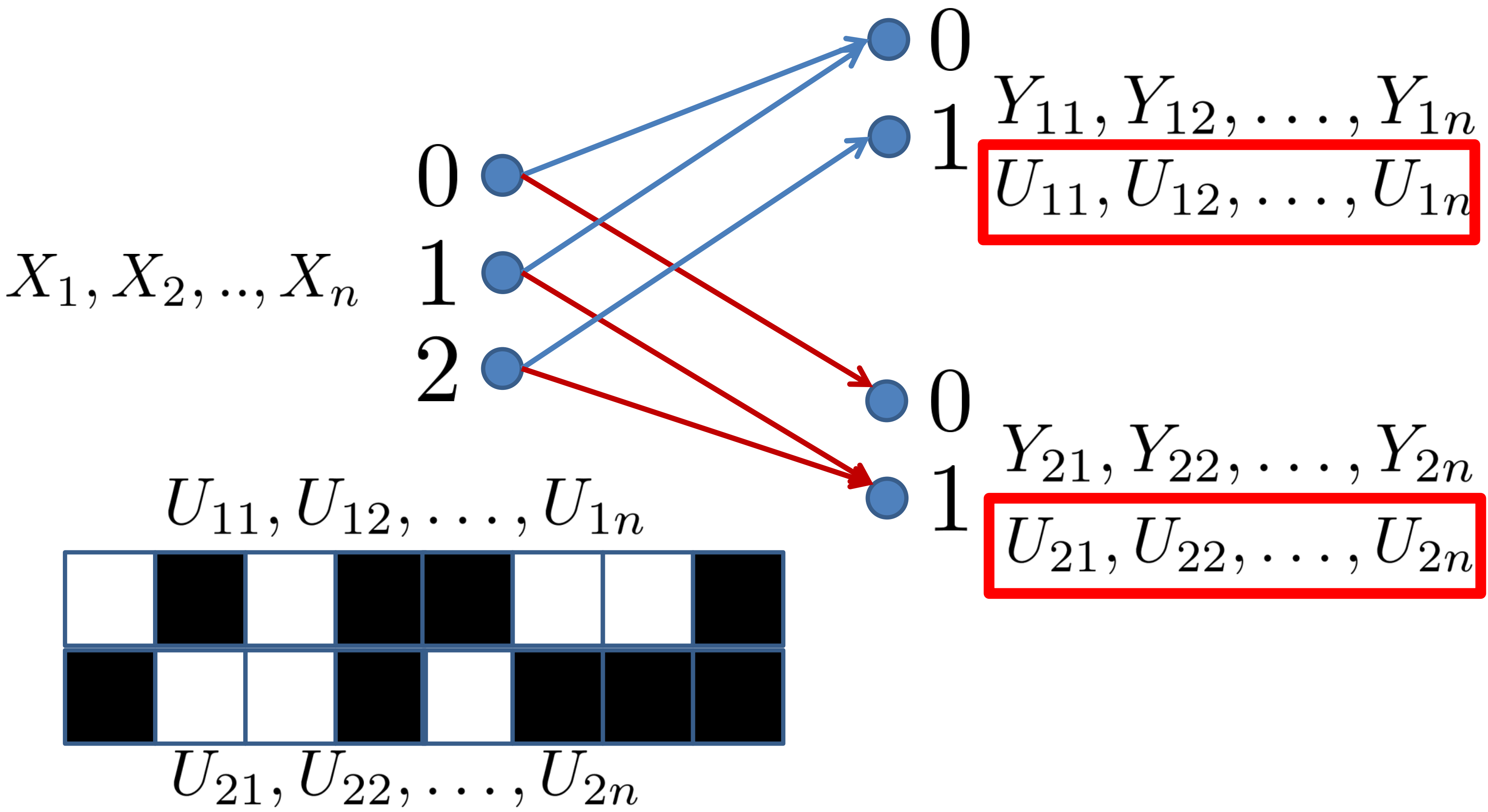
Sketch: Polar Code For Deterministic Broadcast



Sketch: Polar Code For Deterministic Broadcast



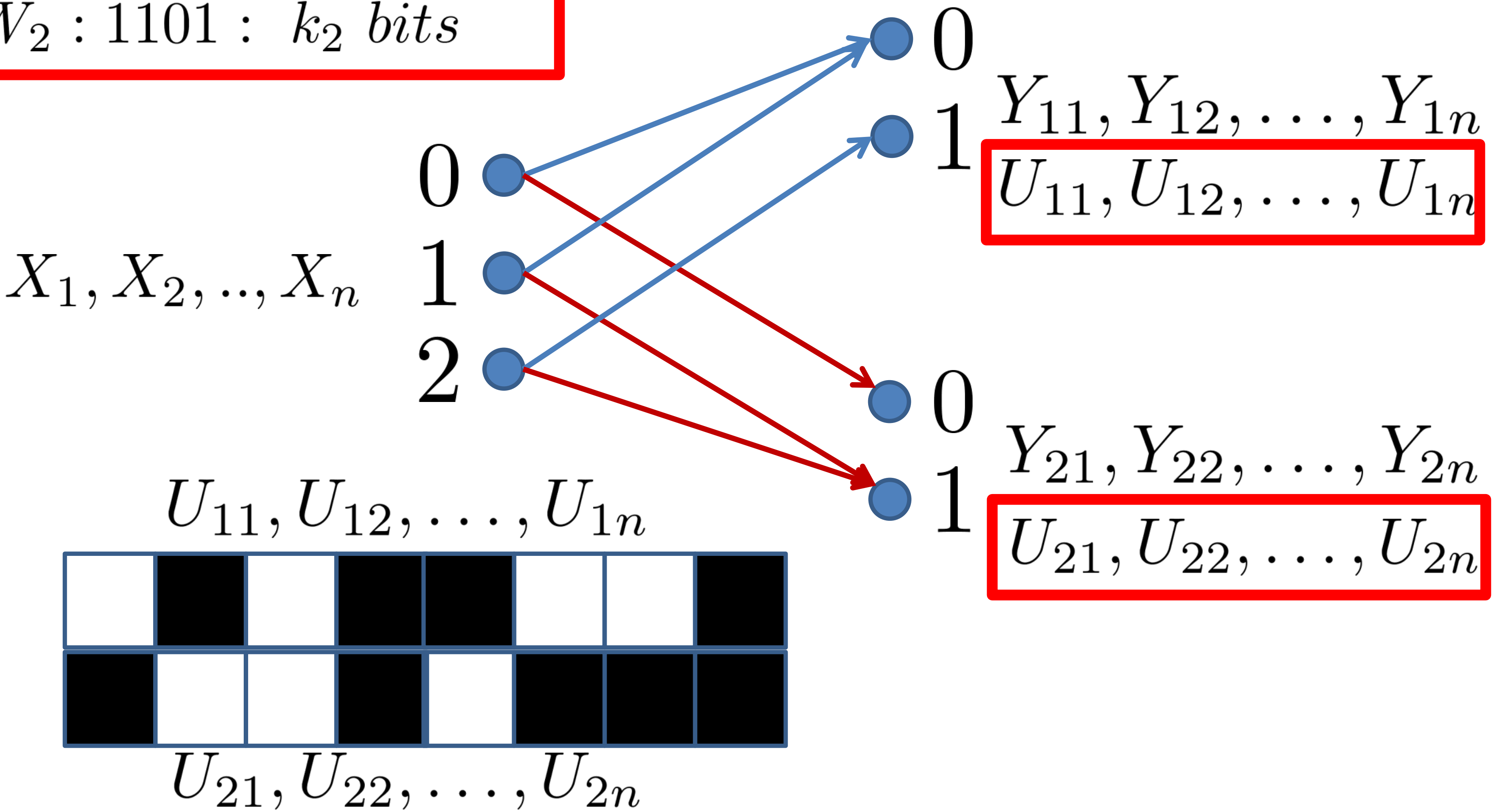
Sketch: Polar Code For Deterministic Broadcast



Sketch: Polar Code For Deterministic Broadcast

$W_1 : 0100111 : \ k_1 \ bits$

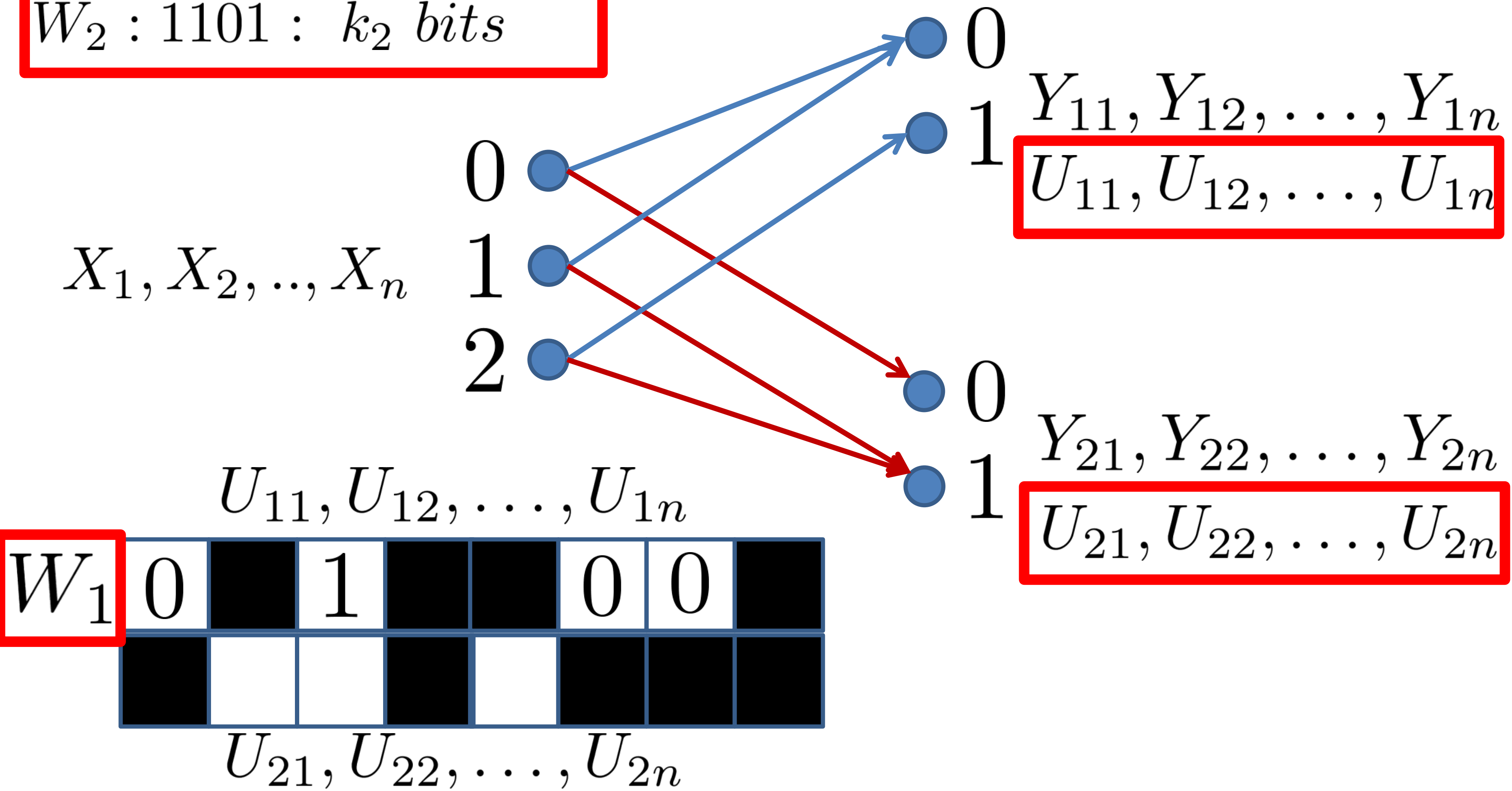
$W_2 : 1101 : \ k_2 \ bits$



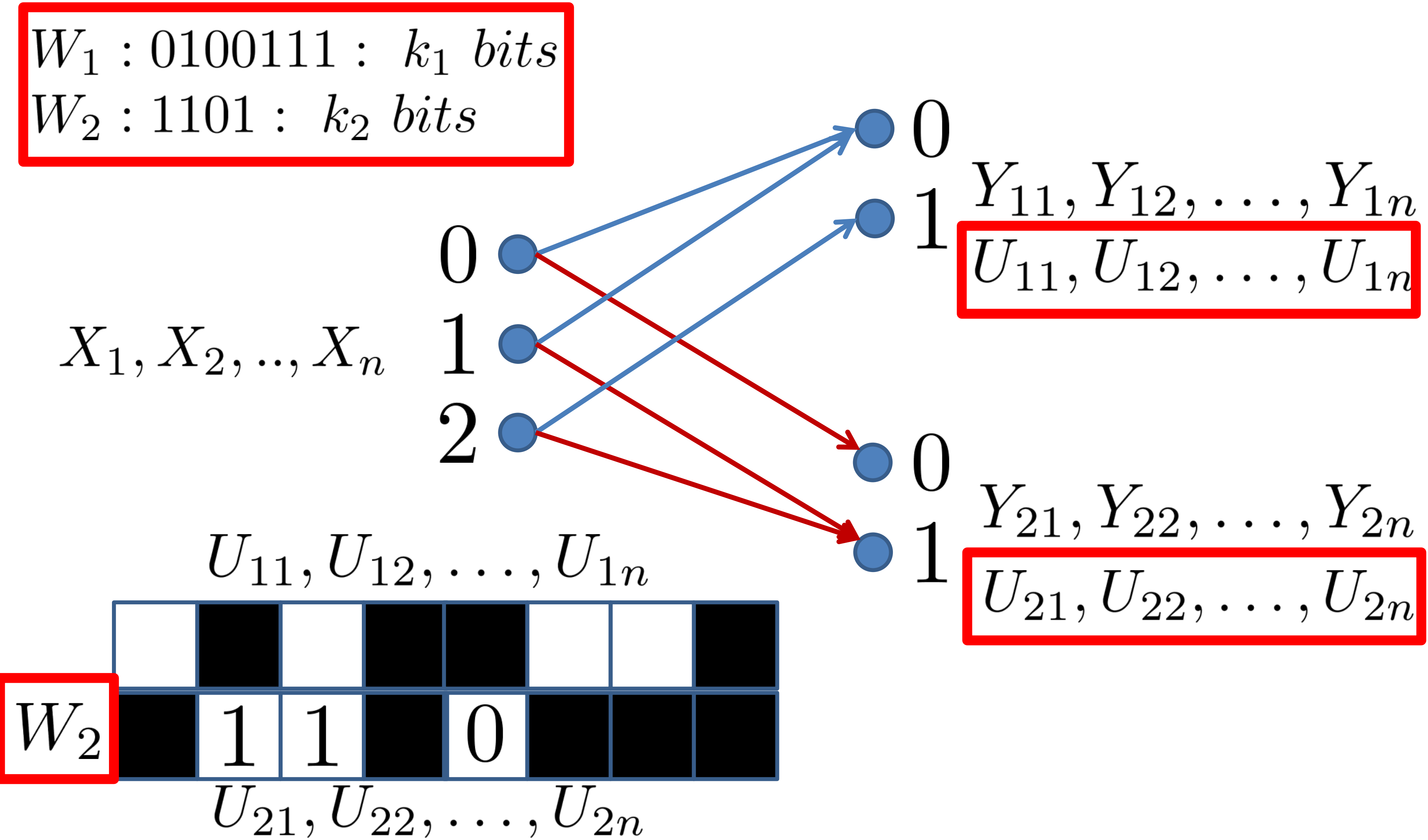
Sketch: Polar Code For Deterministic Broadcast

$W_1 : 0100111 : \ k_1 \ bits$

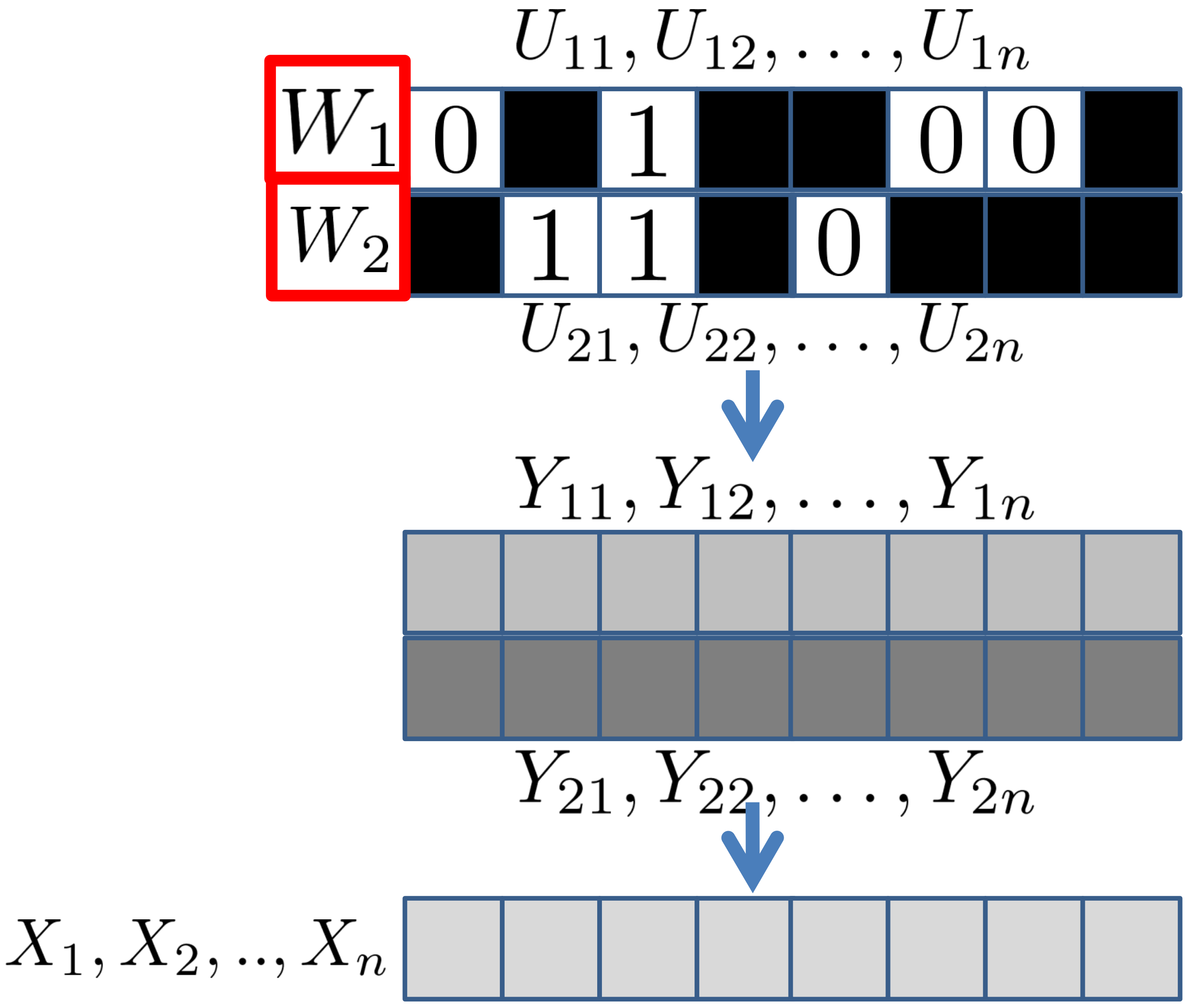
$W_2 : 1101 : \ k_2 \ bits$



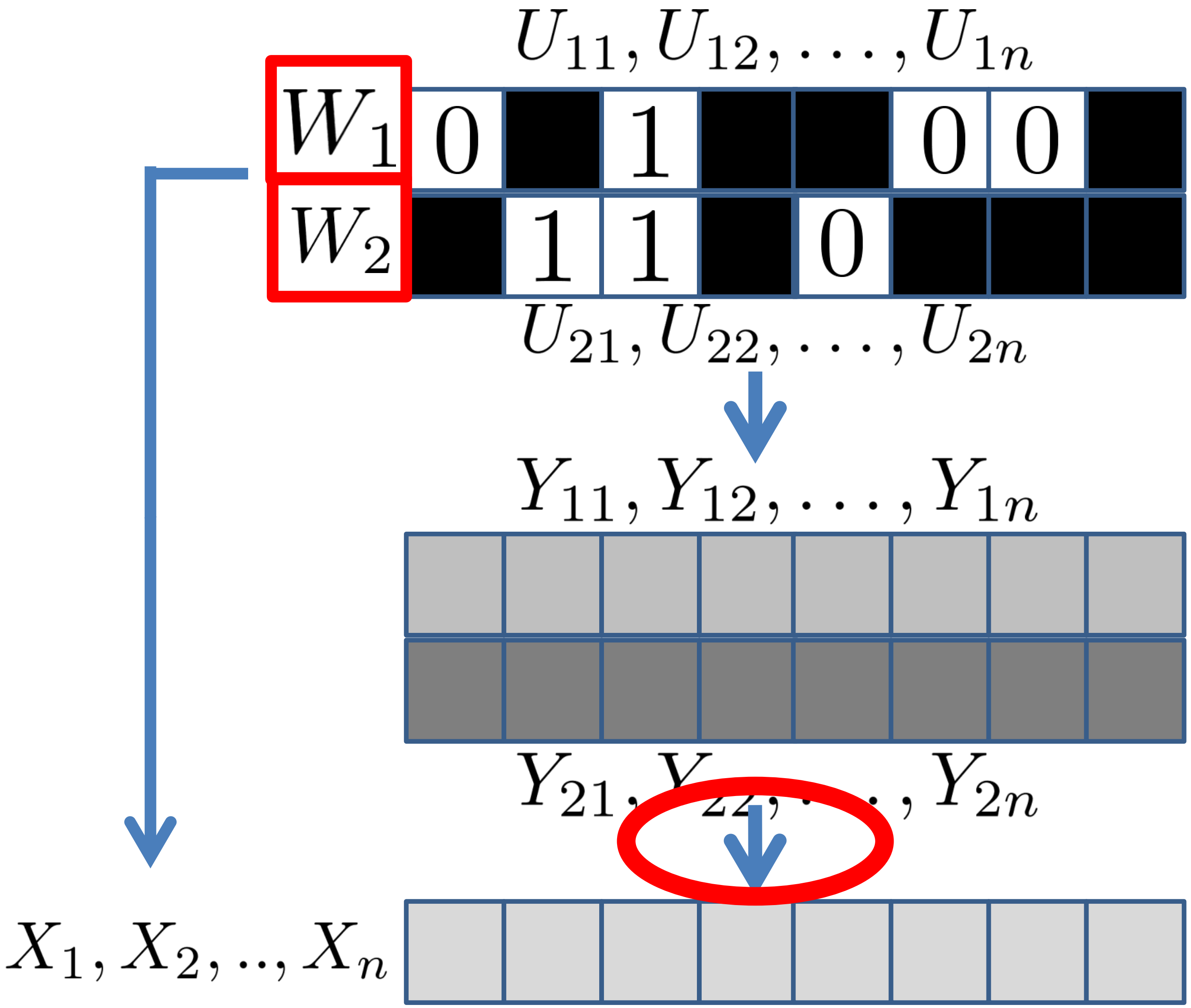
Sketch: Polar Code For Deterministic Broadcast



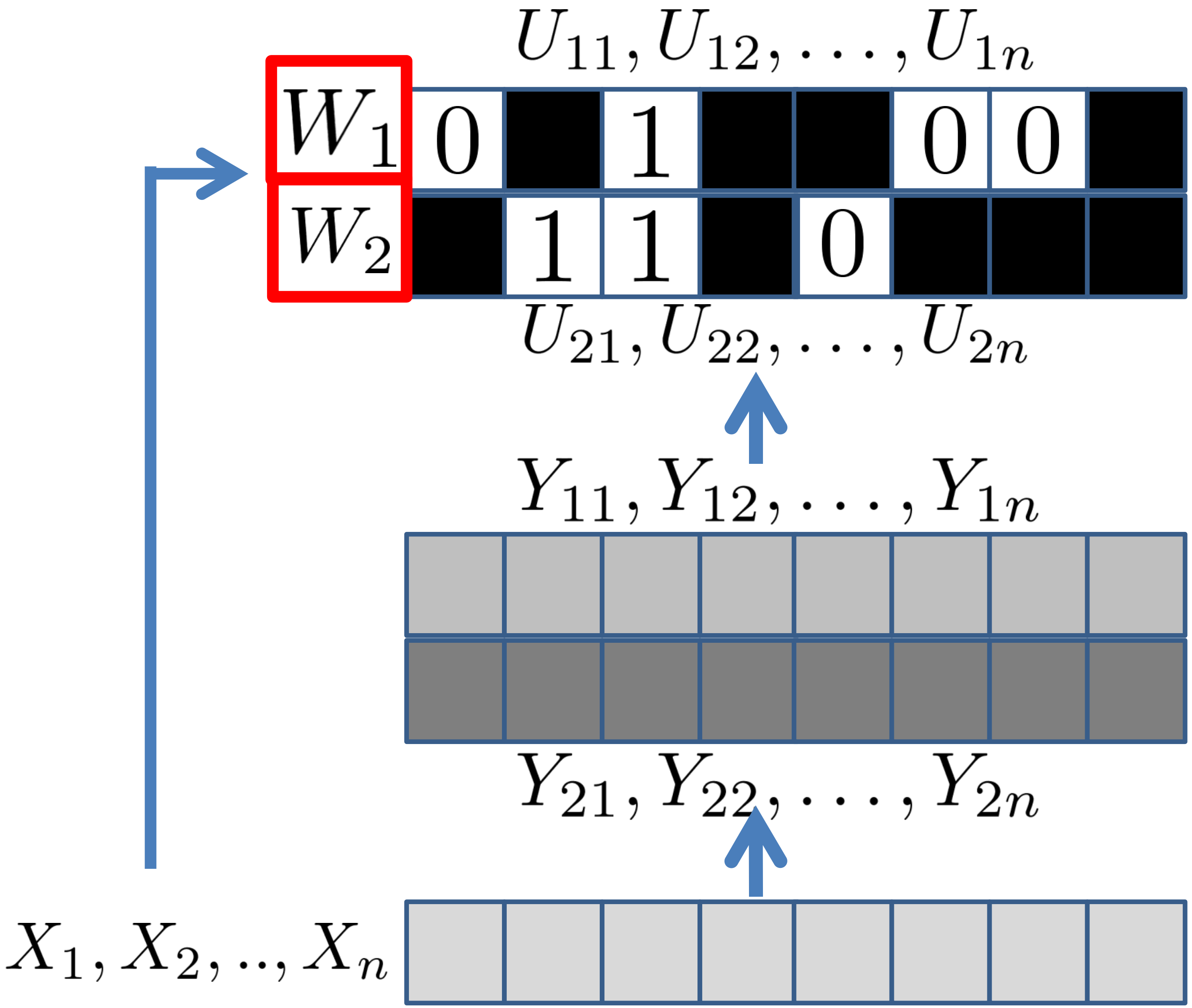
Sketch: Polar Code For Deterministic Broadcast



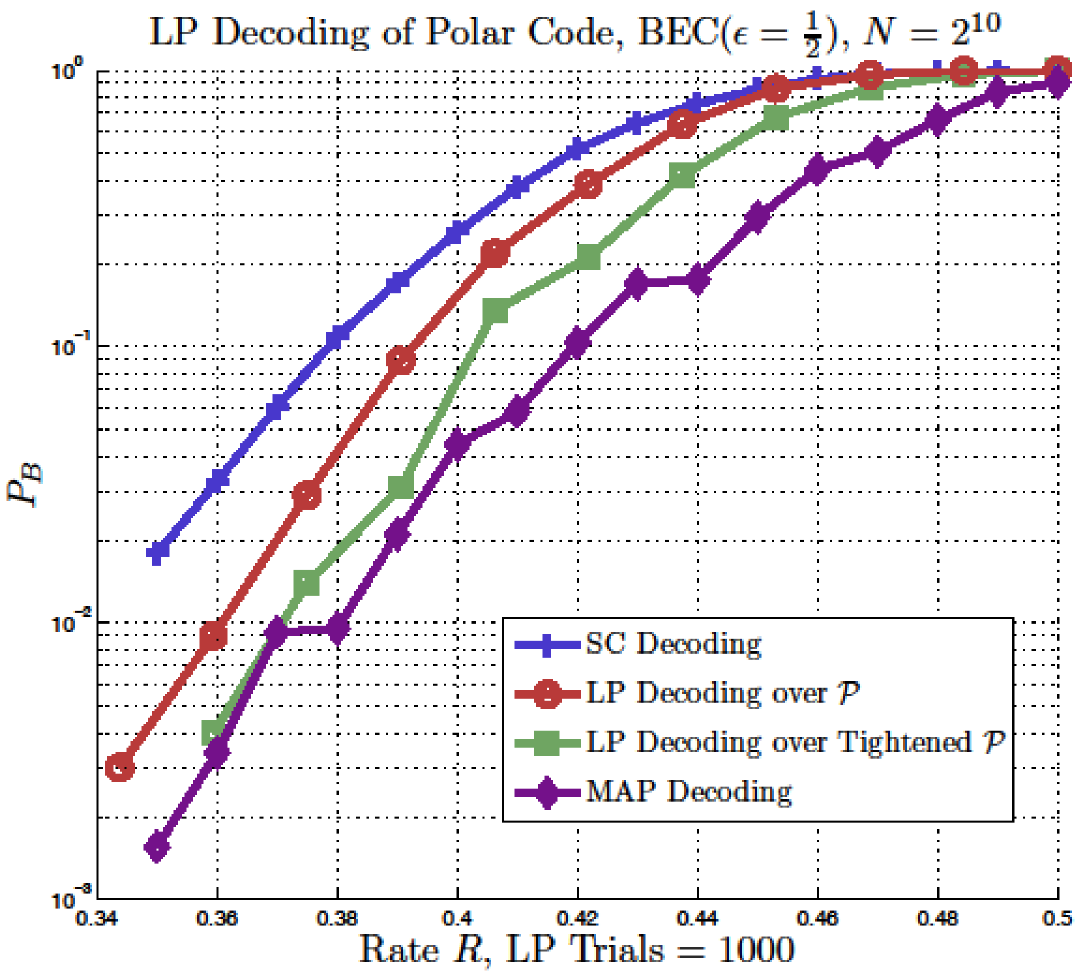
Sketch: Polar Code For Deterministic Broadcast



Sketch: Polar Code For Deterministic Broadcast



Probability of Error Curves Look Like This…



Material Presented From Following References

N.G., Abbe, Gastpar, "Polar Codes For Broadcast Channels," IEEE ISIT Istanbul, Turkey, 2013.

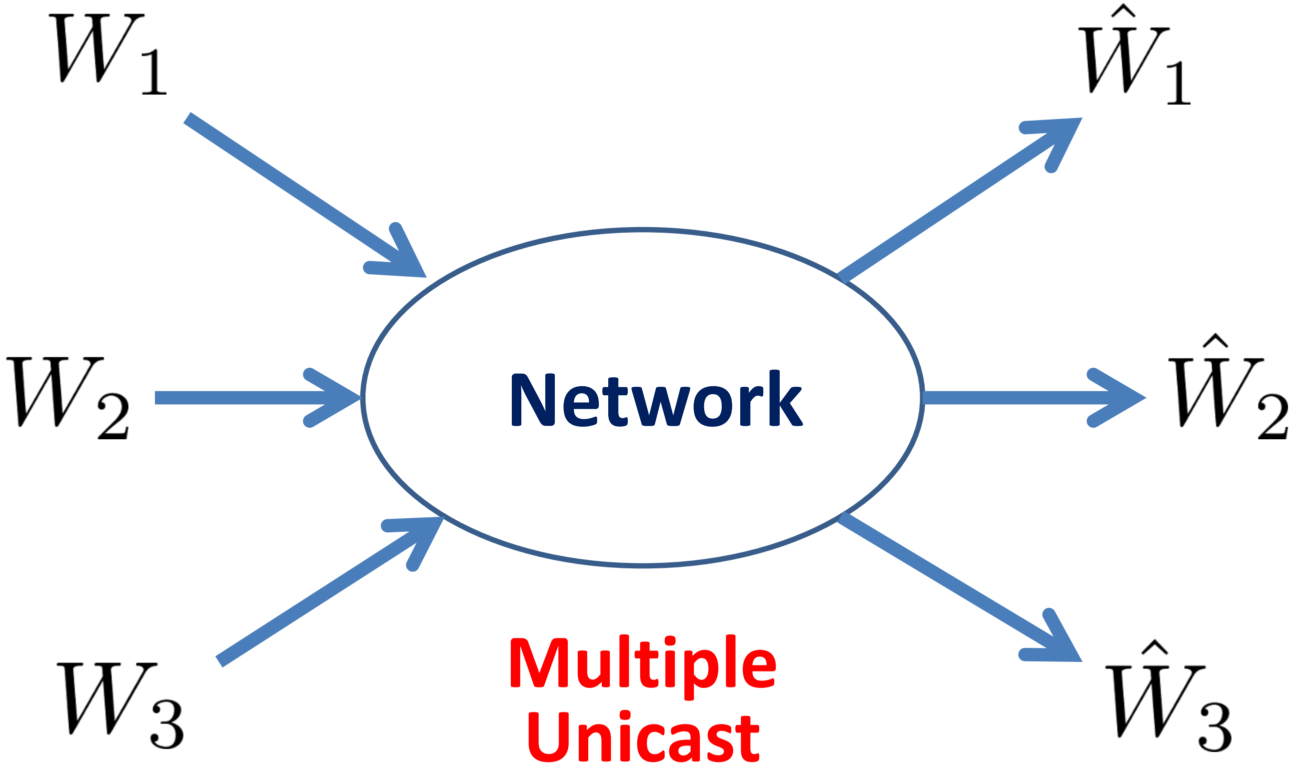
N.G., Abbe, Gastpar, "Polar Codes For Broadcast Channels," Submission to IEEE IT Transactions, Reference: http://arxiv.org/abs/1301.6150, 2013.

N.G., Abbe, Gastpar, "Polar Codes For Deterministic Broadcast Channels," International Zurich Seminar on Communications, Switzerland, 2012.

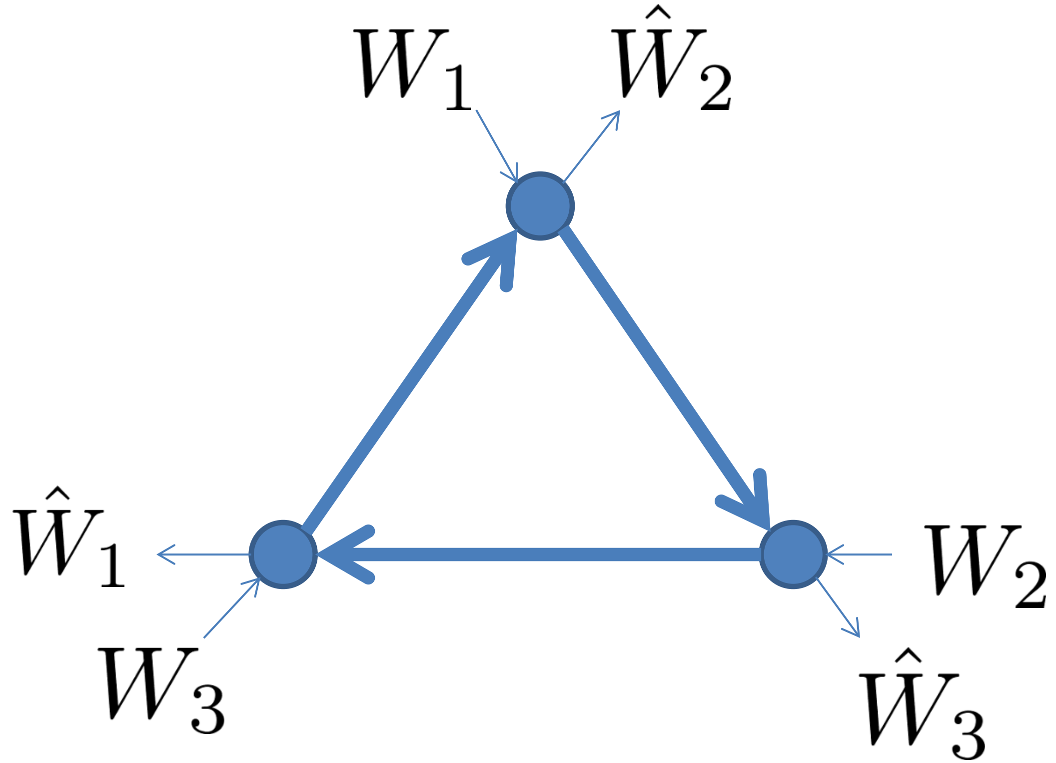
N.G., Korada, Gastpar, "On LP Decoding Of Polar Codes," IEEE ITW, Dublin, Ireland, 2010.

Part II
A Simple Noiseless Interfering Network

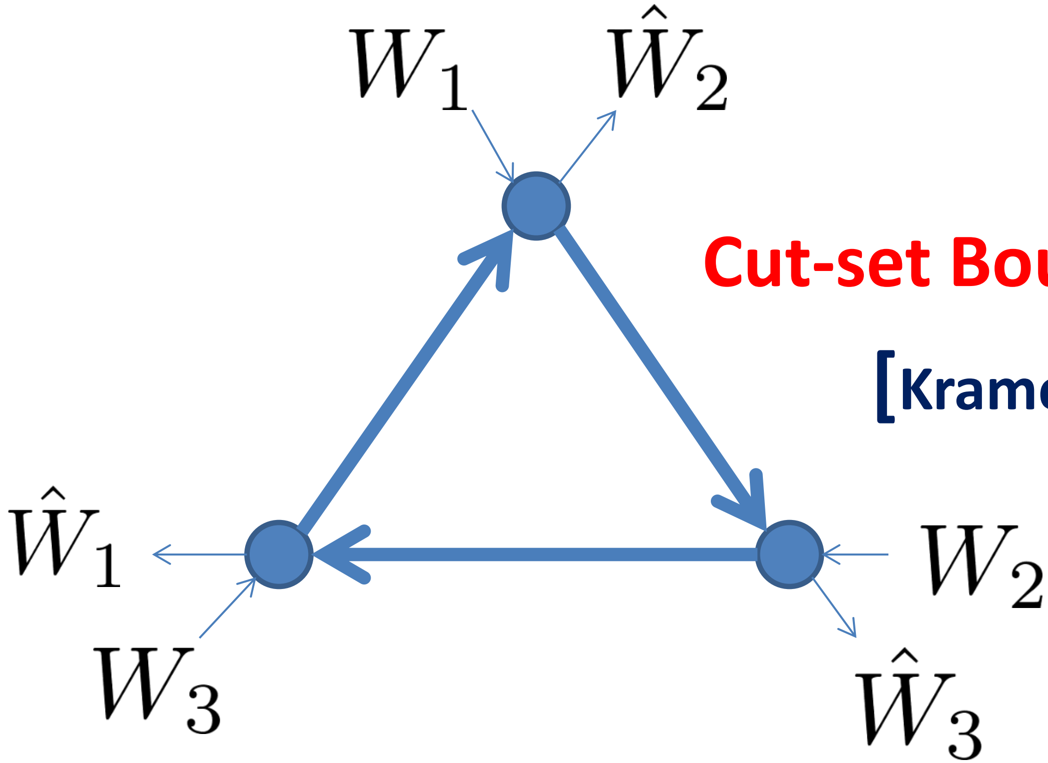
Communication and Computation in Networks



Communication and Computation in Networks



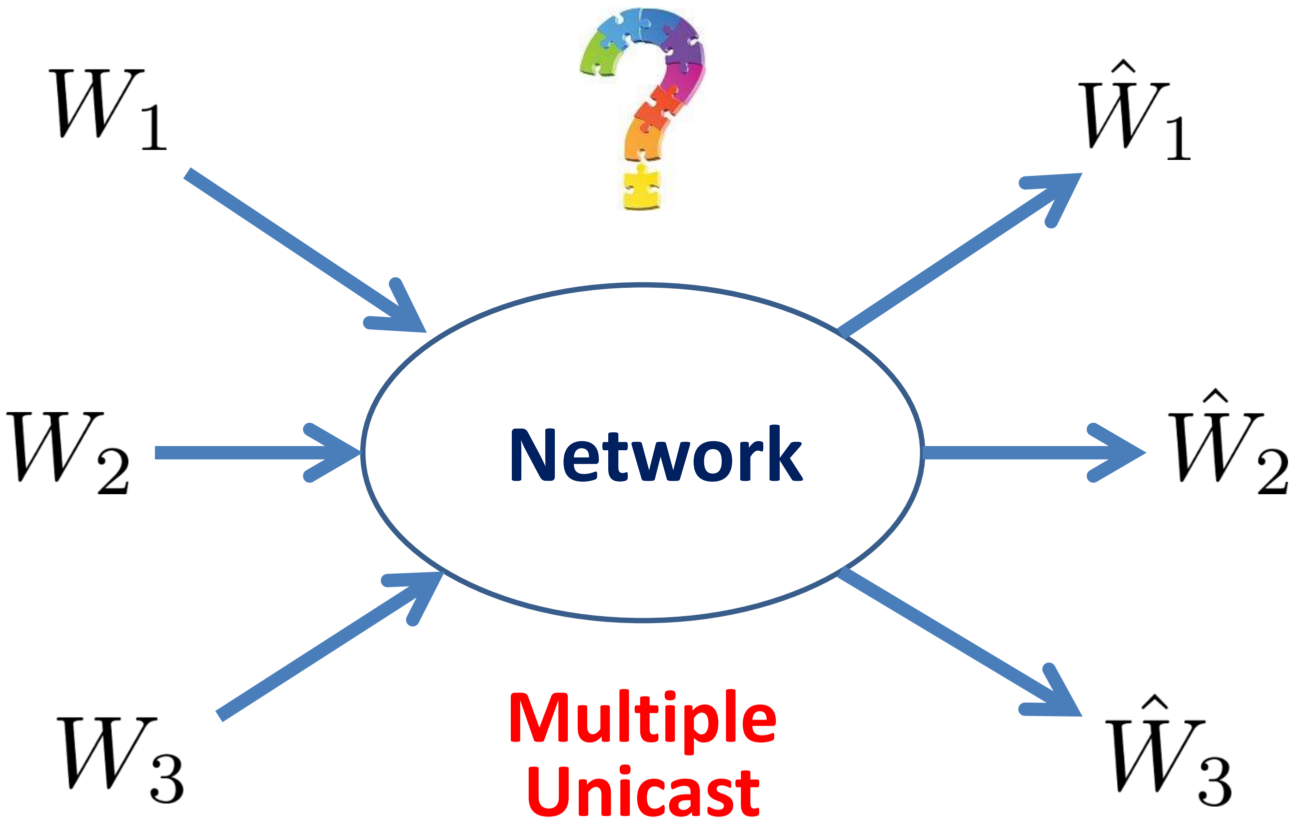
Communication and Computation in Networks



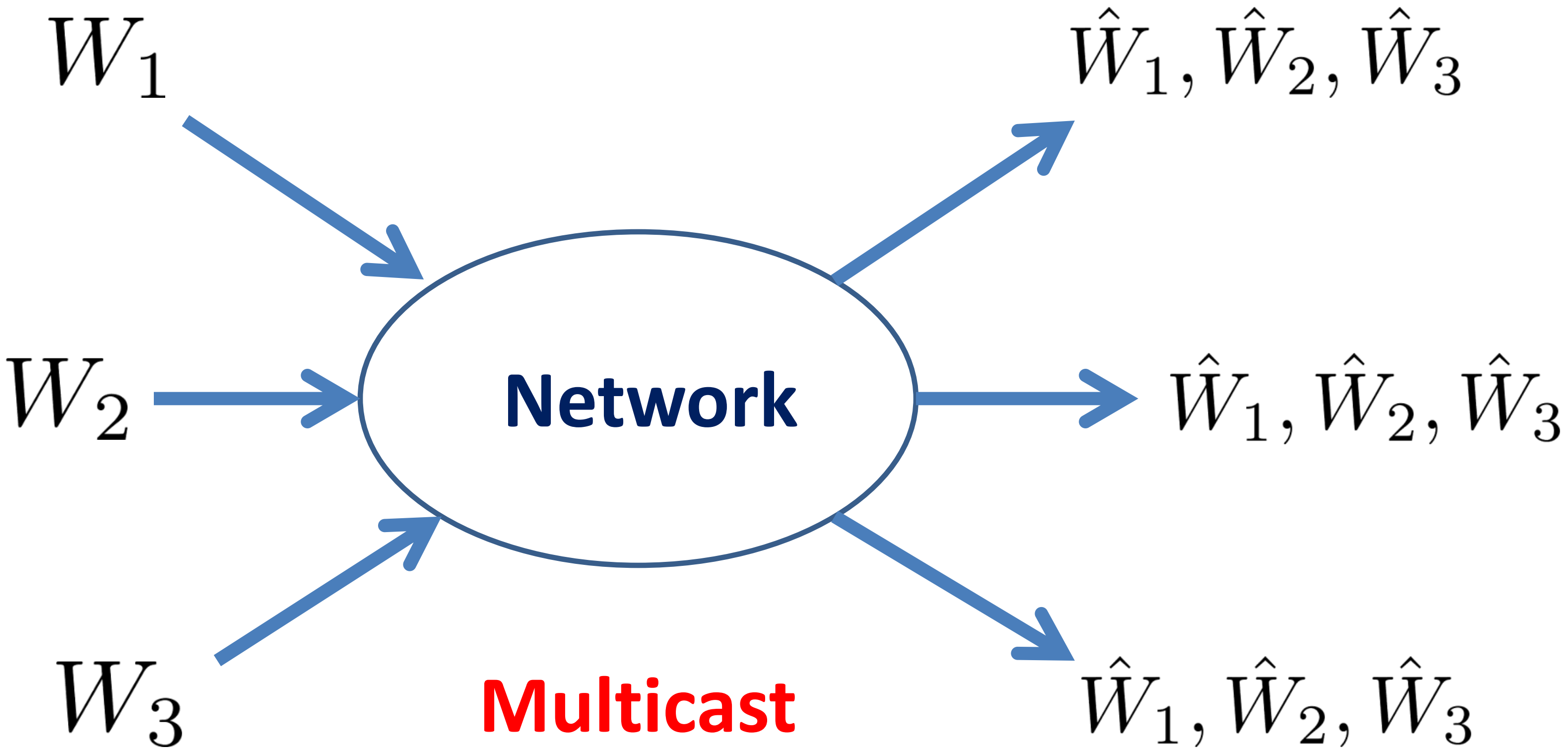
Cut-set Bound Not Tight

[KramerSavari2006]

Communication and Computation in Networks

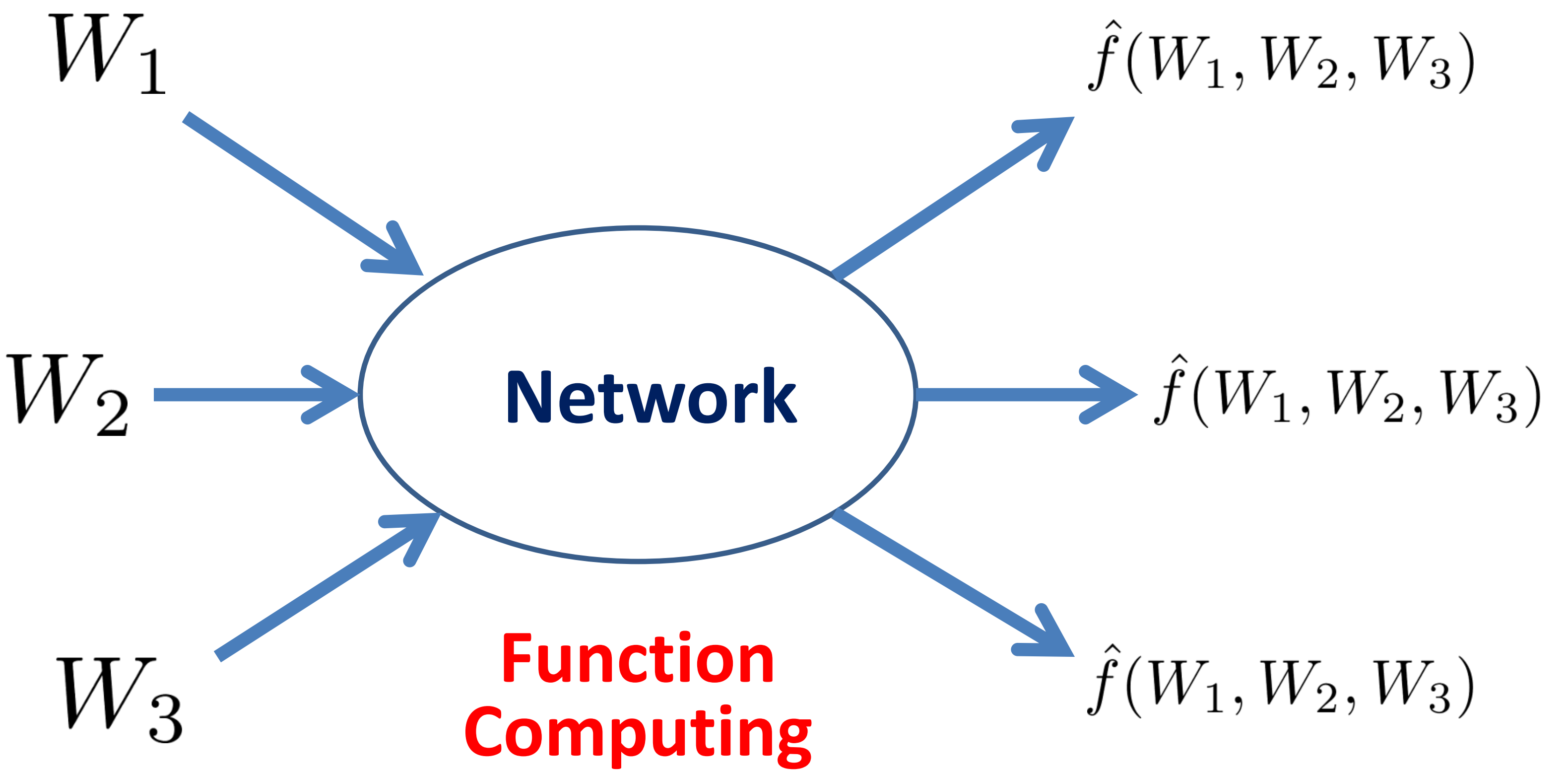


Communication and Computation in Networks

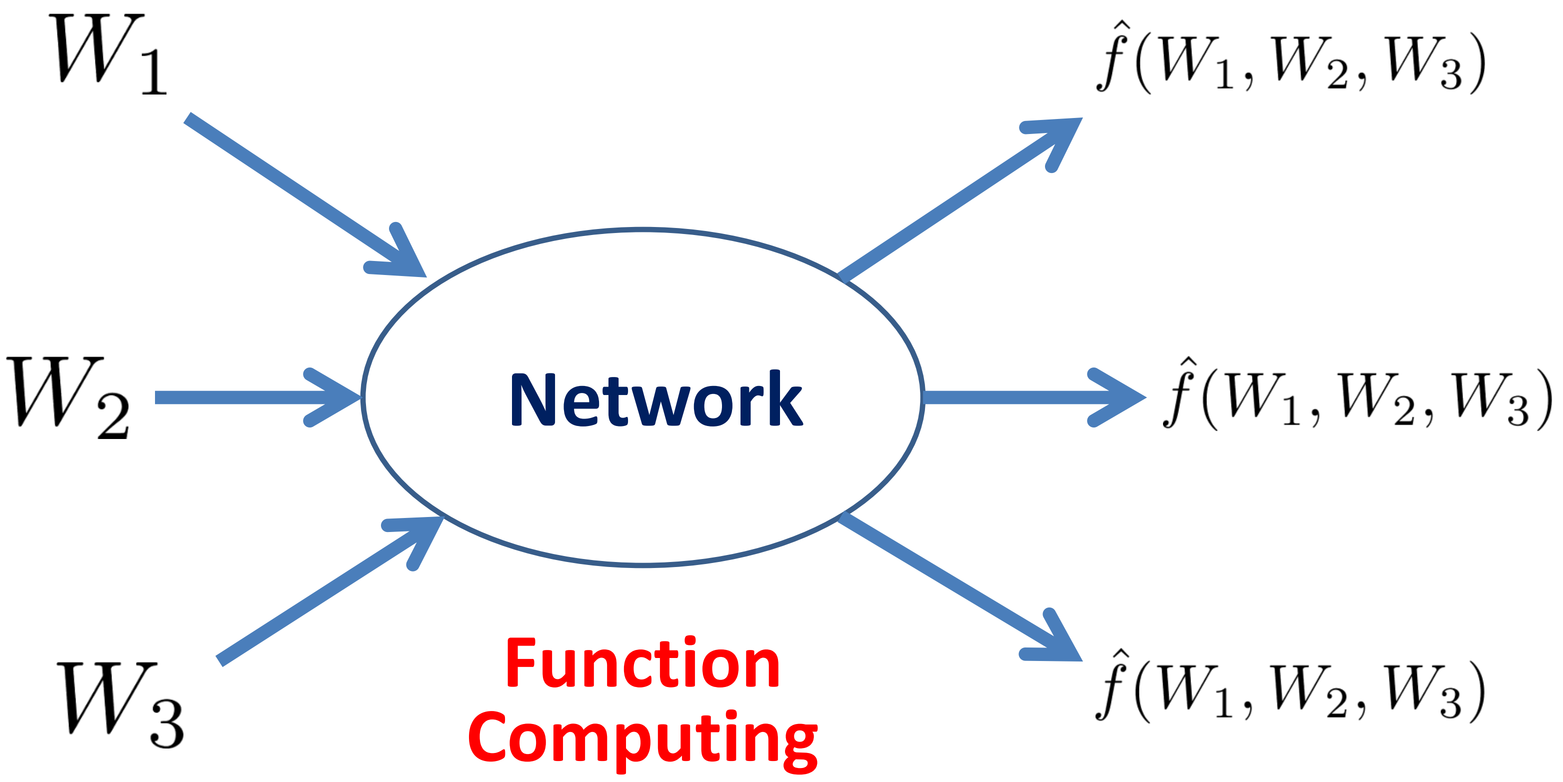


[ACLY2000][YLYC2003][KM2003][JSCEEJT2005][HMKKESL2006]

Communication and Computation in Networks

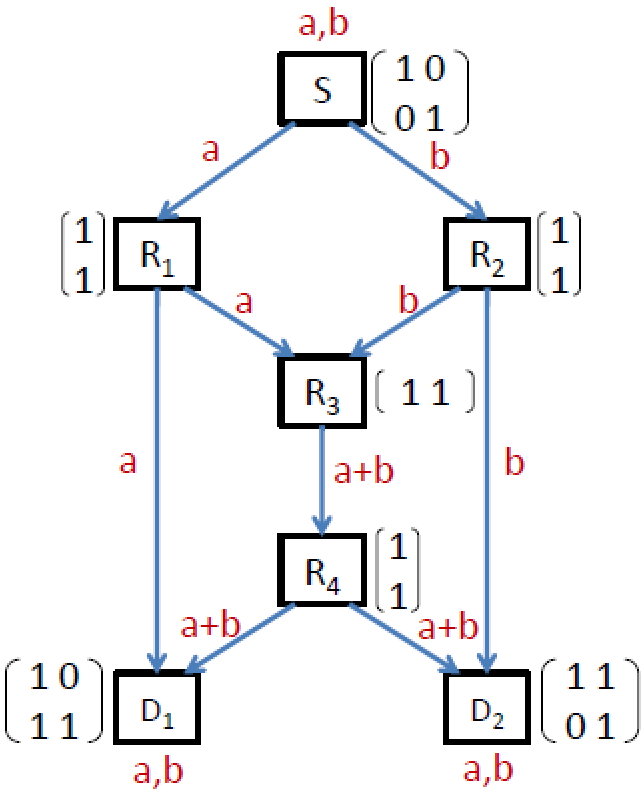


Communication and Computation in Networks



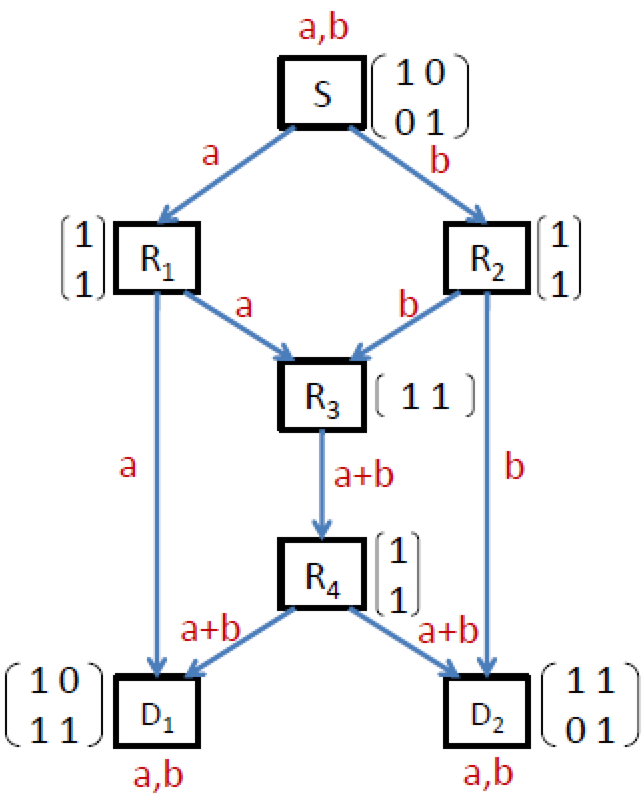
[RL2010][RD2012][AFKZ2011][NG2011]

Communication and Computation in Networks

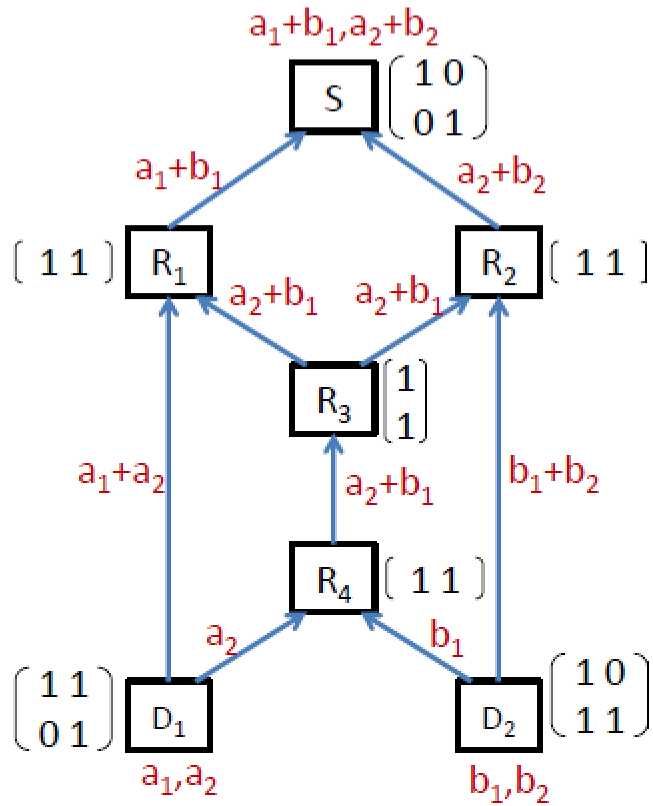


Multicast Communication

Communication and Computation in Networks

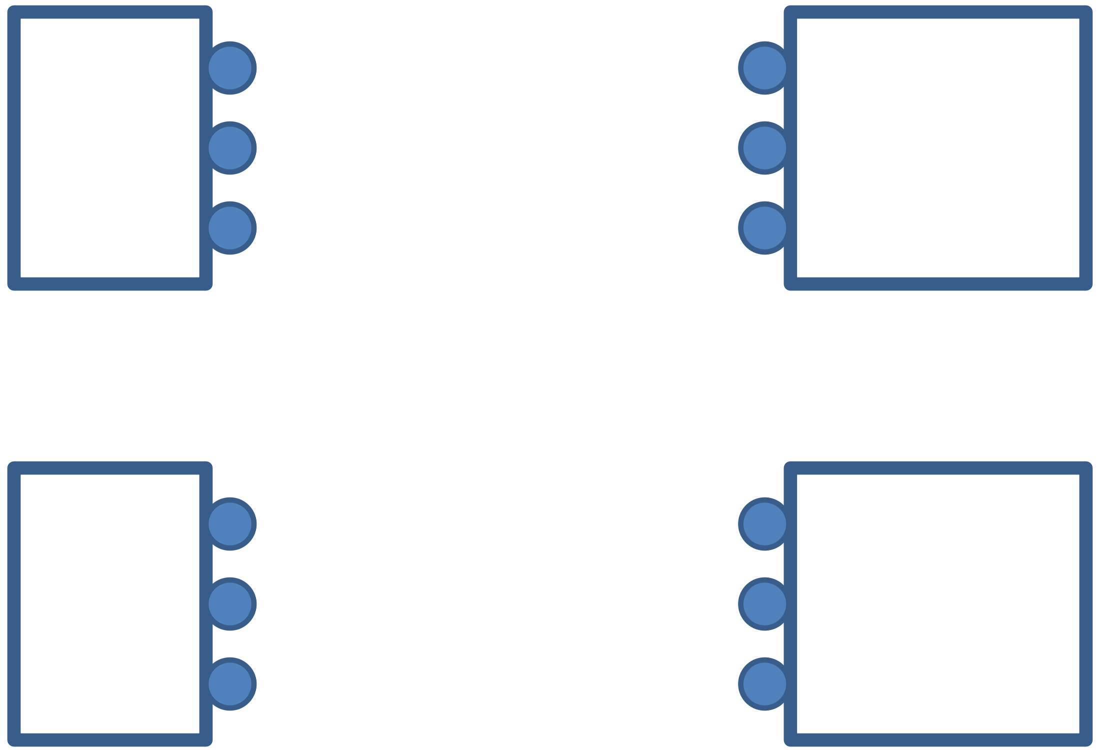


Multicast Communication

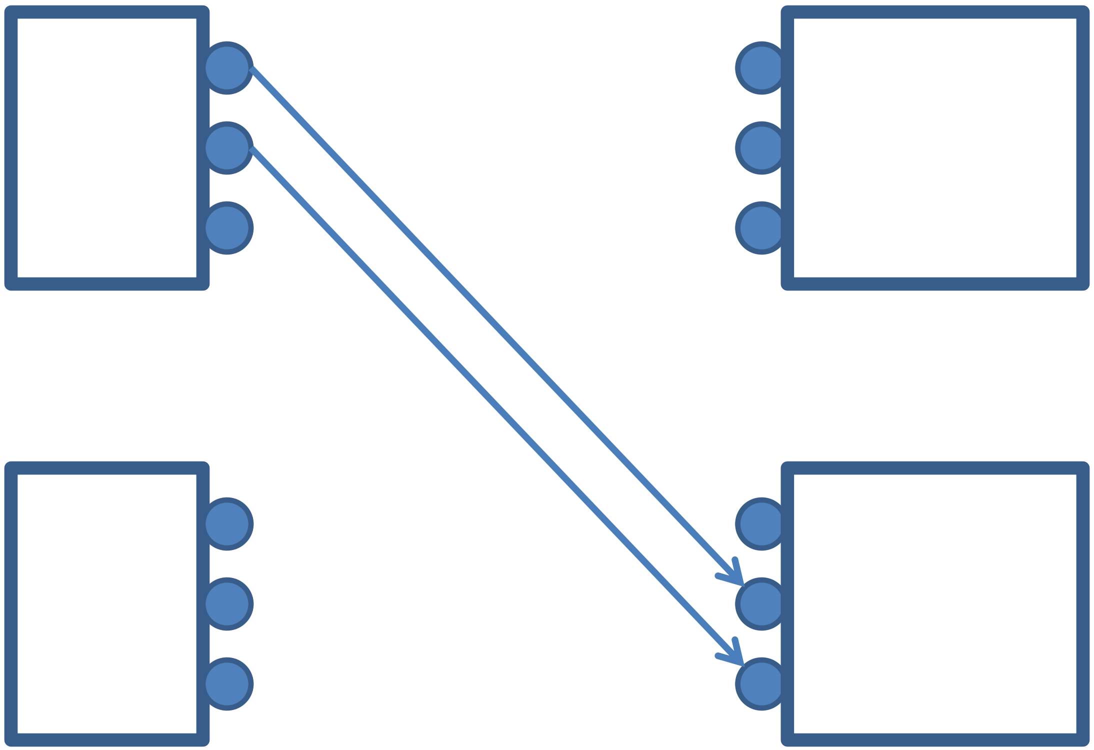


Computing in Single-Receiver Network

Countably Infinite Class of (m, L) Networks

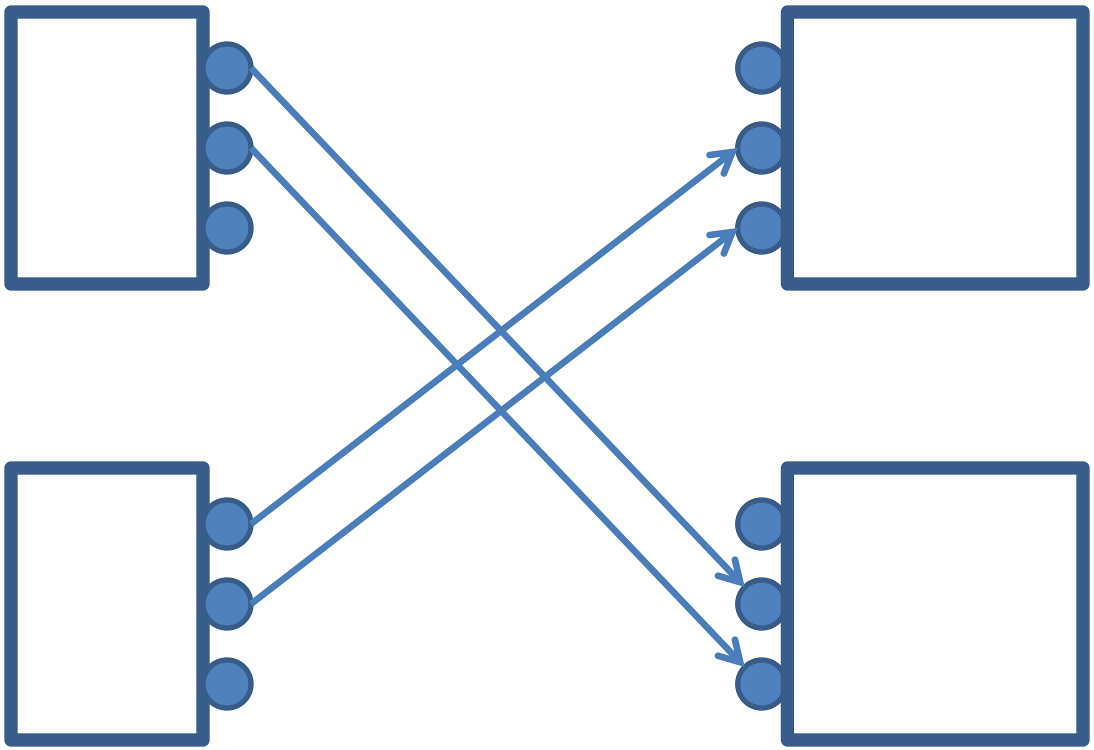


Countably Infinite Class of (m, L) Networks



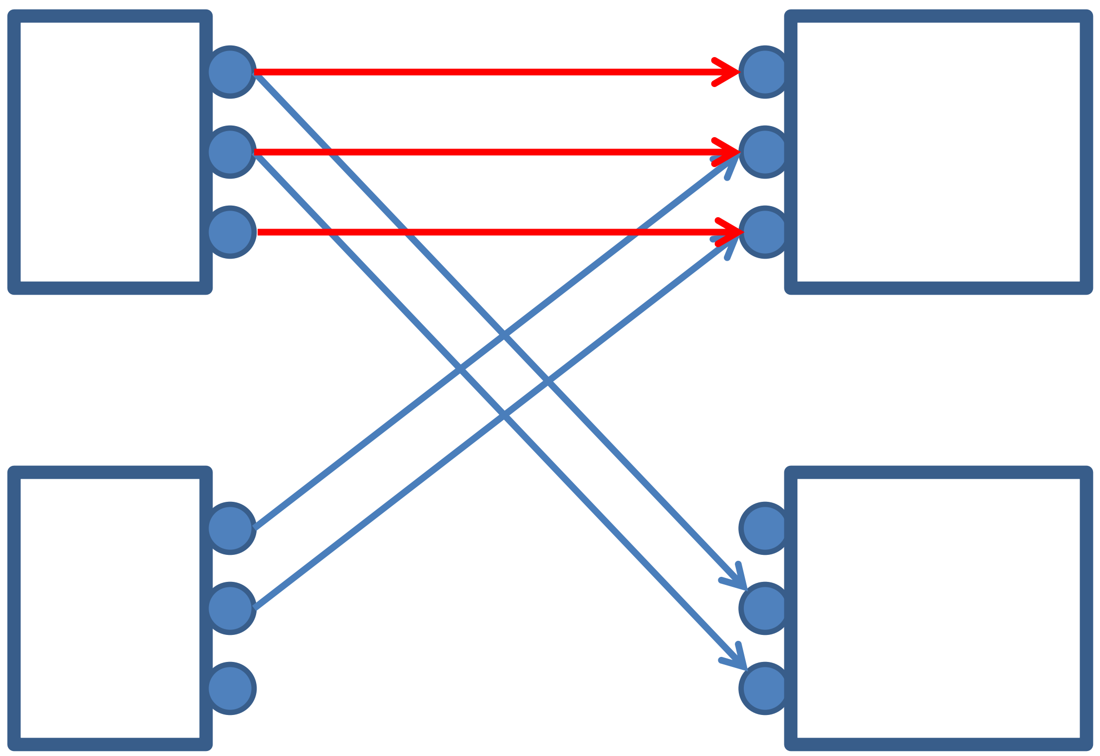
$$m = 2$$

Countably Infinite Class of (m, L) Networks



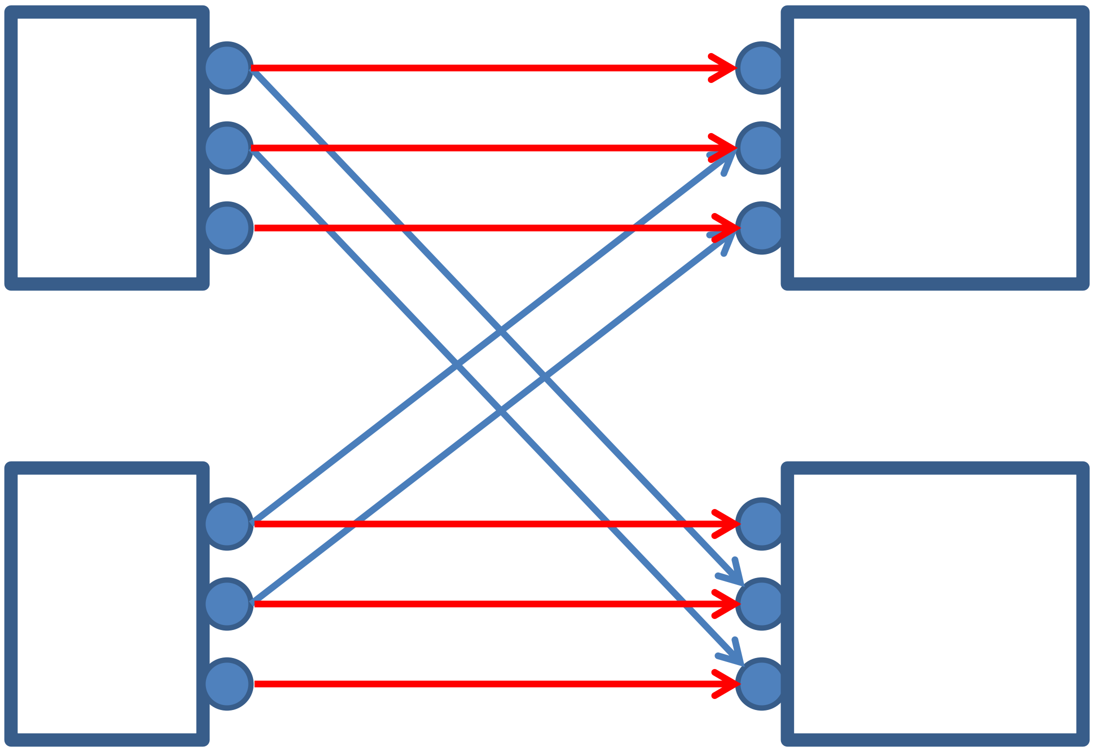
$$m = 2$$

Countably Infinite Class of (m, L) Networks



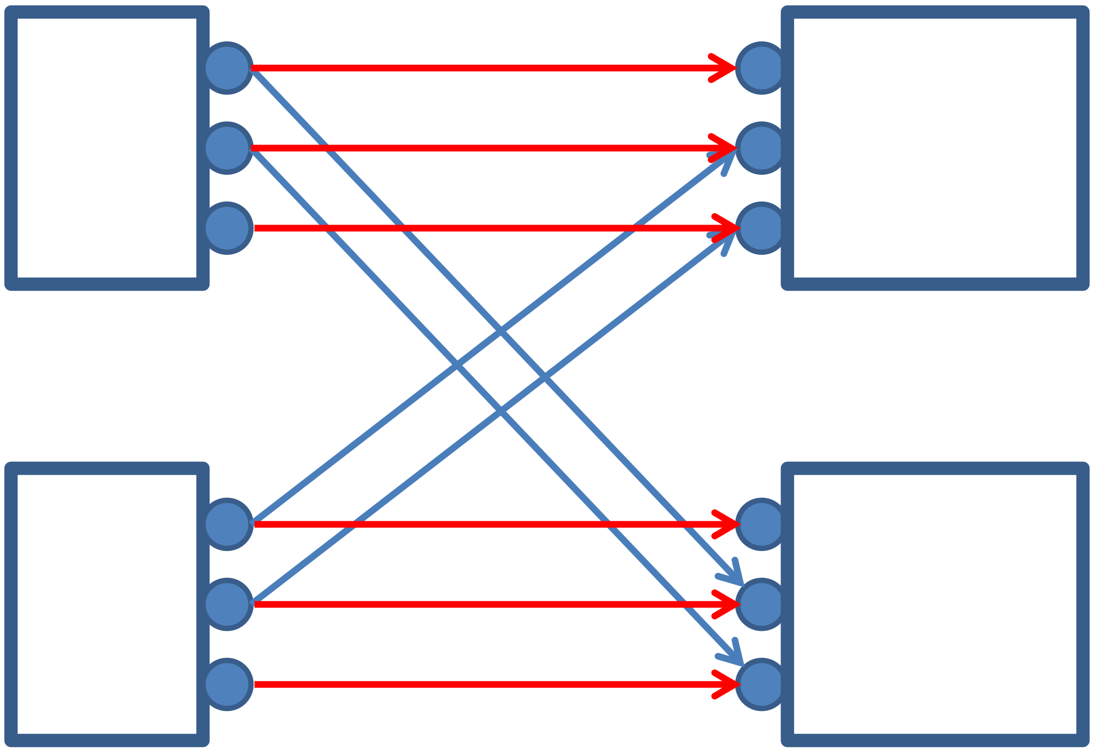
$$m = 2$$

Countably Infinite Class of (m, L) Networks



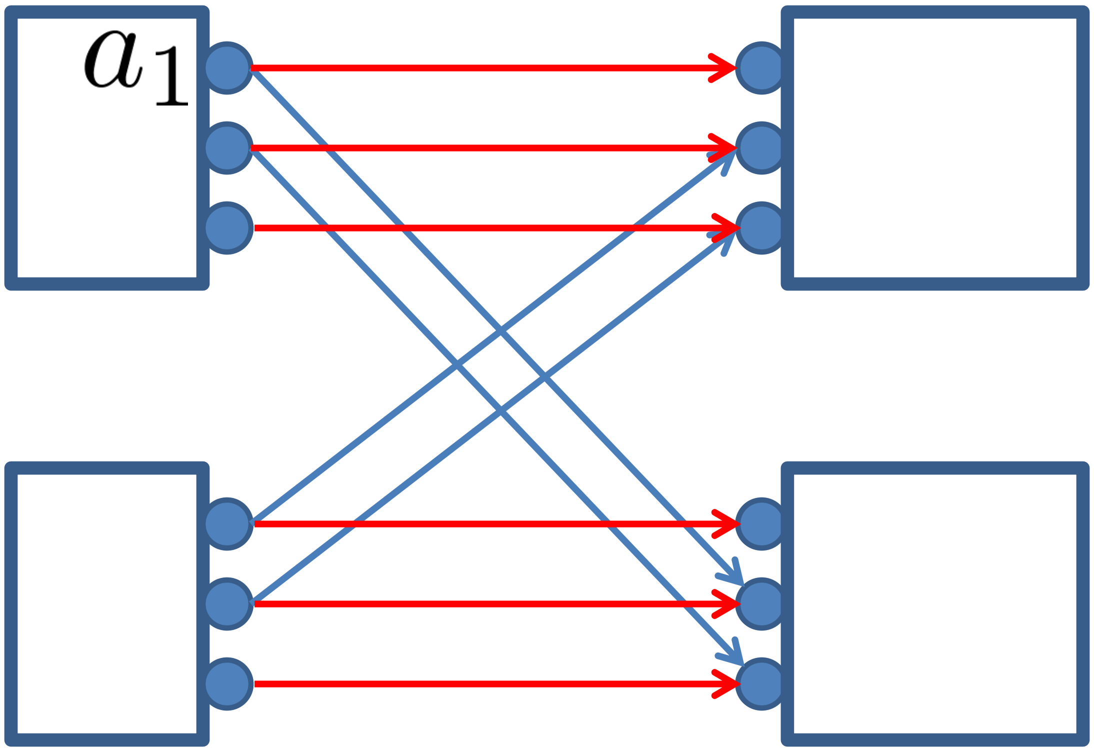
$$m = 2, L = 3$$

Countably Infinite Class of (m, L) Networks



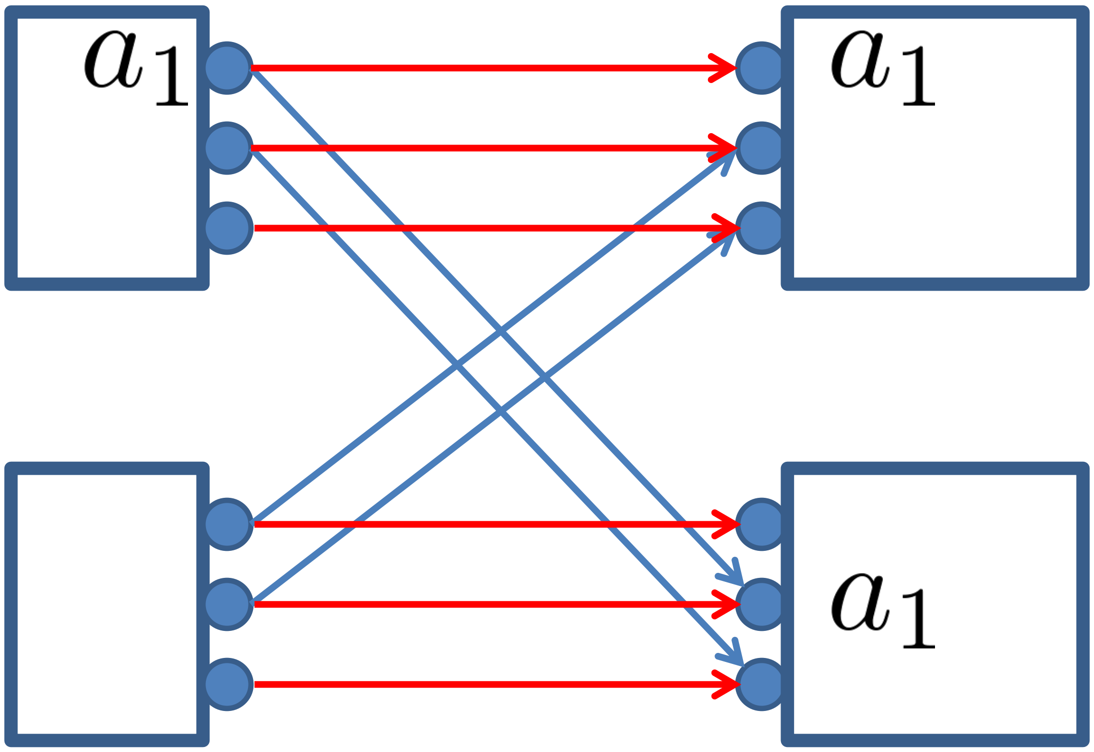
$$m = 2, L = 3, \alpha \equiv \frac{m}{L} = \frac{2}{3}$$

Countably Infinite Class of (m, L) Networks



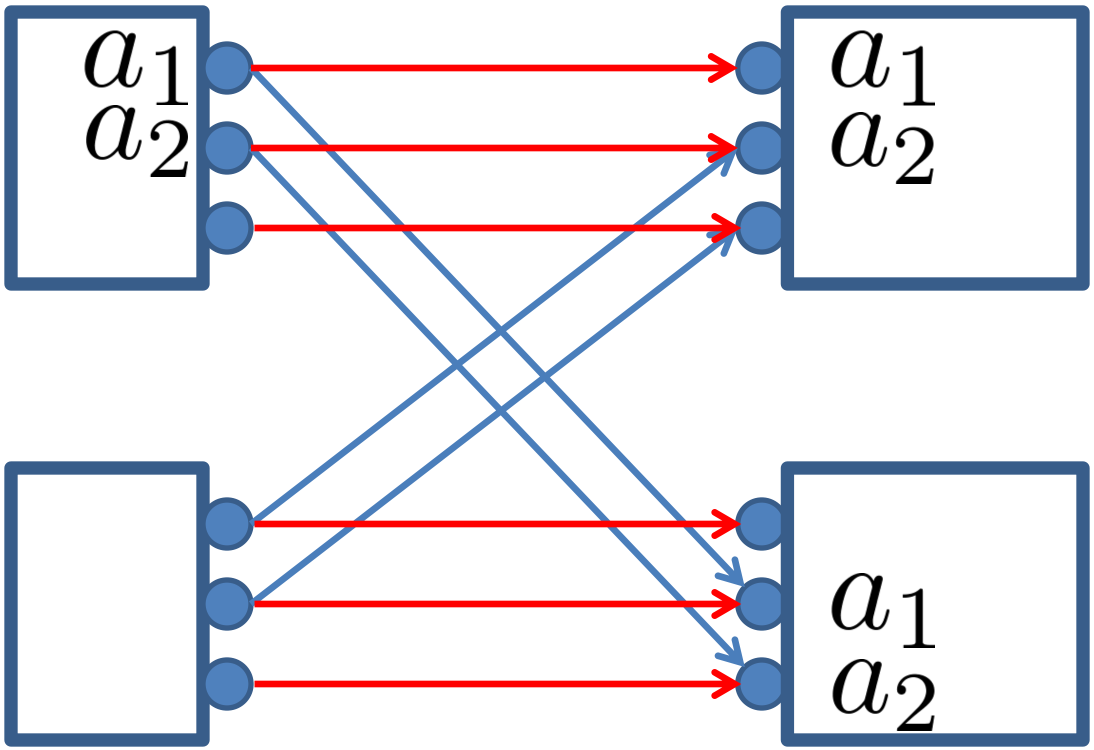
$$m = 2, L = 3, \alpha \equiv \frac{m}{L} = \frac{2}{3}$$

Countably Infinite Class of (m, L) Networks



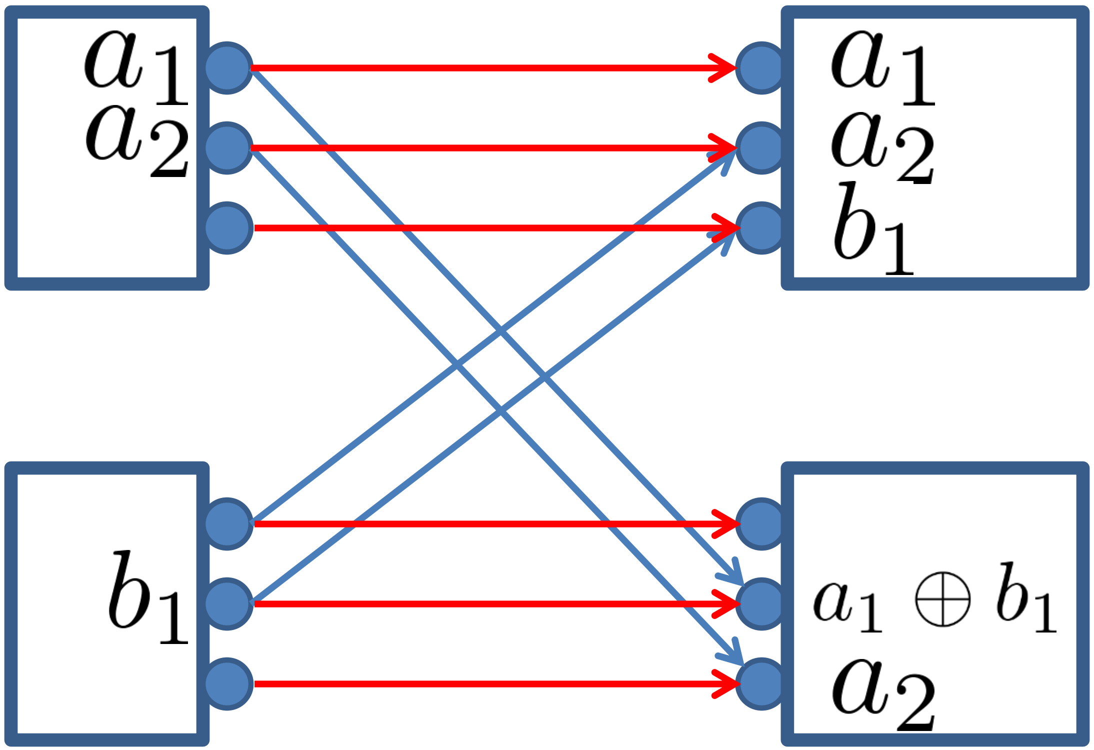
$$m = 2, L = 3, \alpha \equiv \frac{m}{L} = \frac{2}{3}$$

Countably Infinite Class of (m, L) Networks



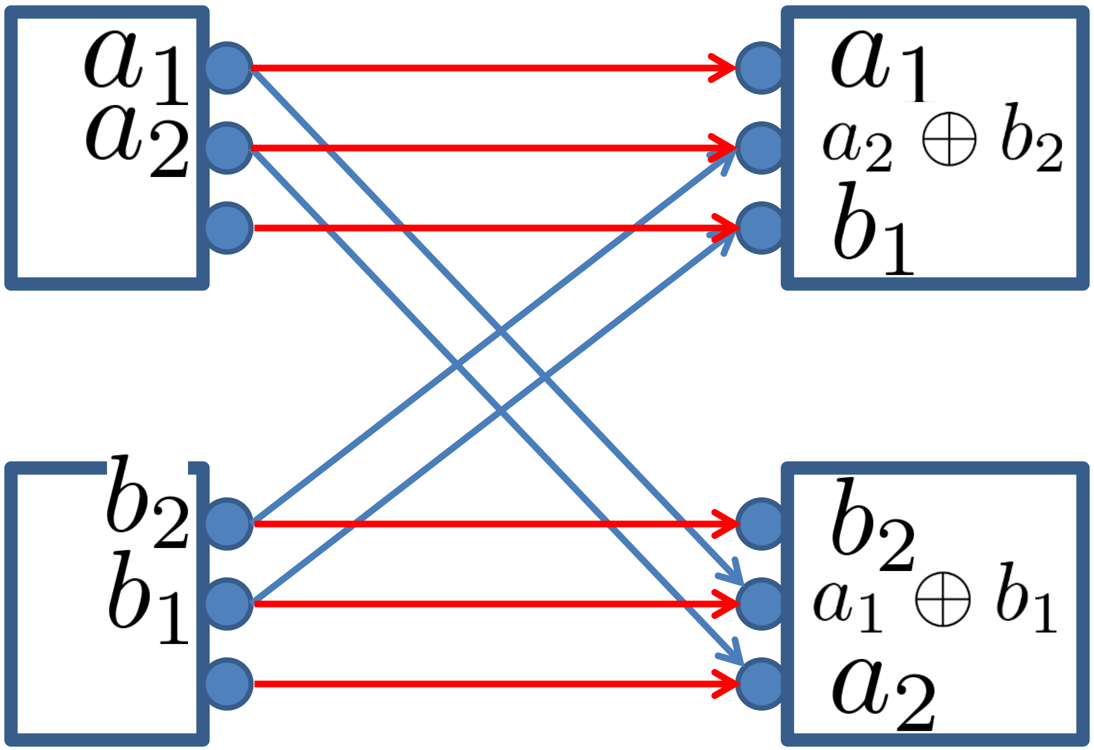
$$m = 2, L = 3, \alpha \equiv \frac{m}{L} = \frac{2}{3}$$

Countably Infinite Class of (m, L) Networks



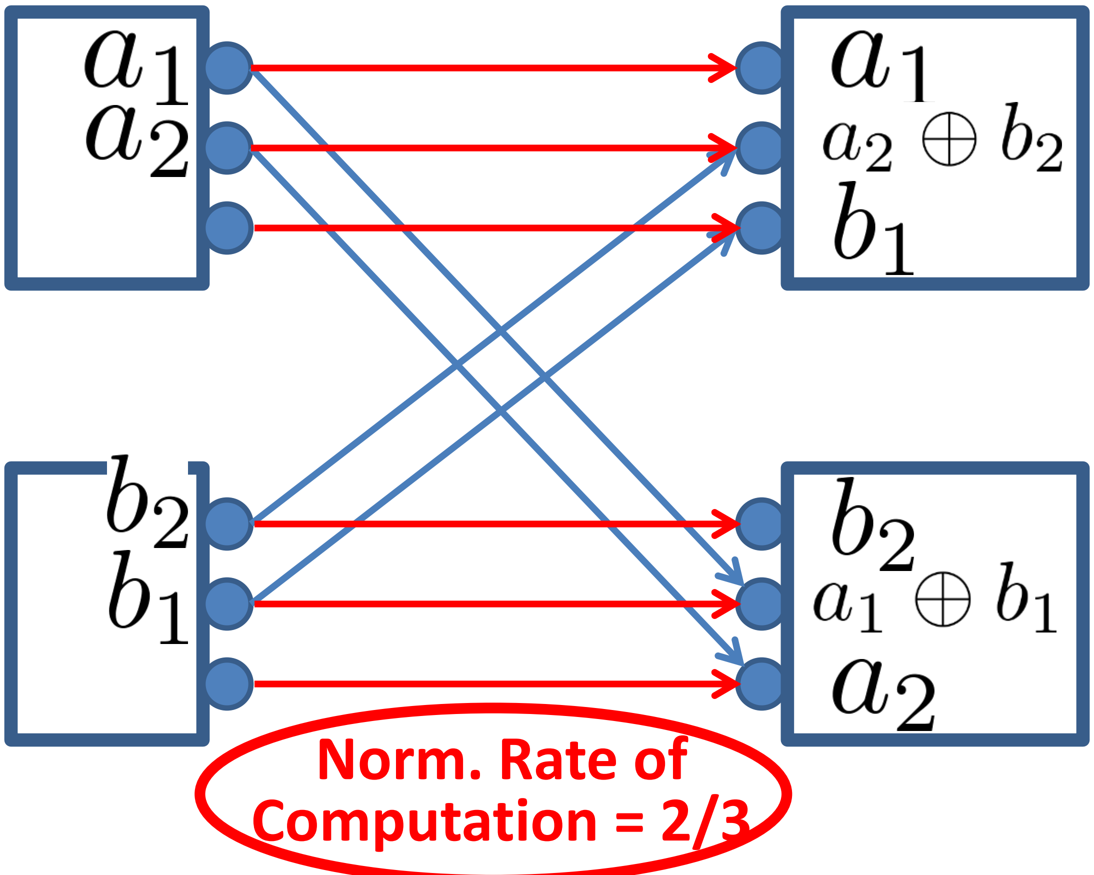
$$m = 2, L = 3, \alpha \equiv \frac{m}{L} = \frac{2}{3}$$

Countably Infinite Class of (m, L) Networks



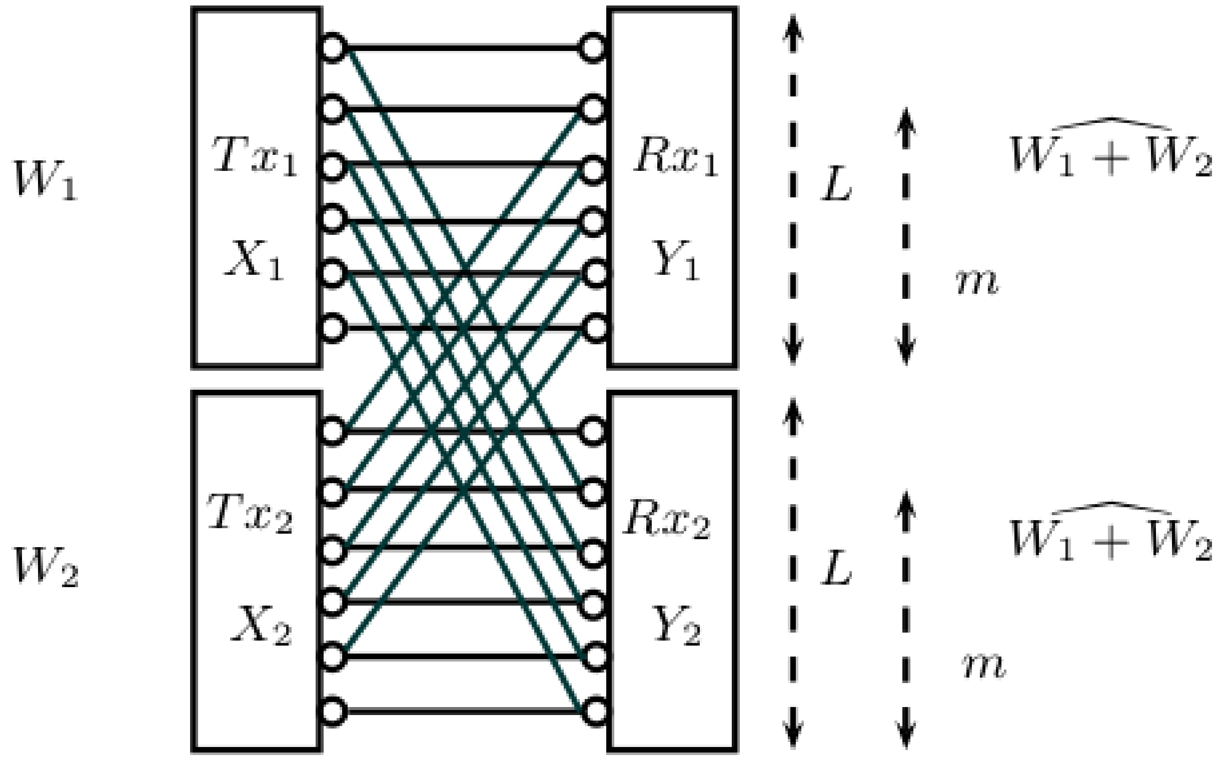
$$m = 2, L = 3, \alpha \equiv \frac{m}{L} = \frac{2}{3}$$

Countably Infinite Class of (m, L) Networks

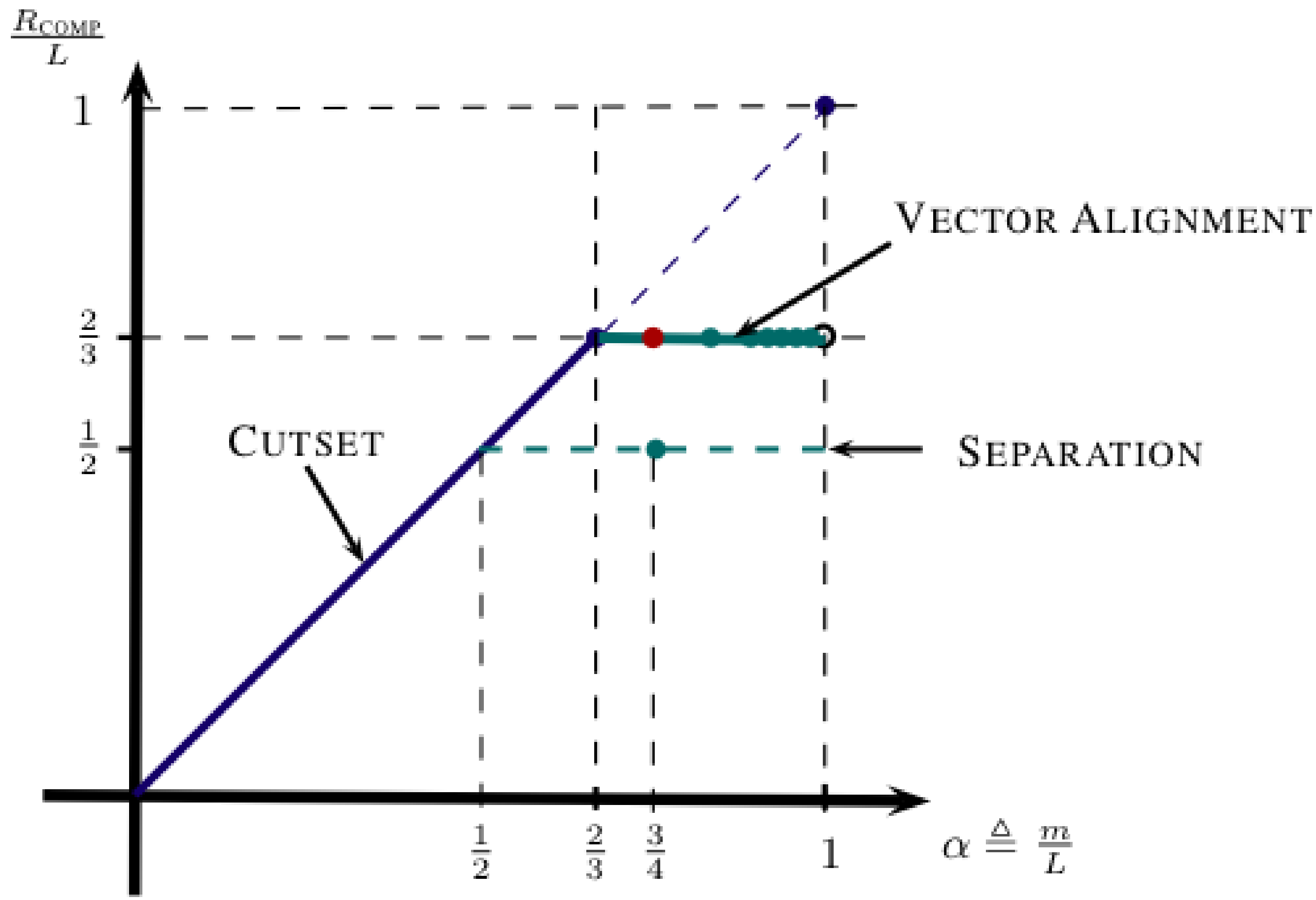


$$m = 2, L = 3, \alpha \equiv \frac{m}{L} = \frac{2}{3}$$

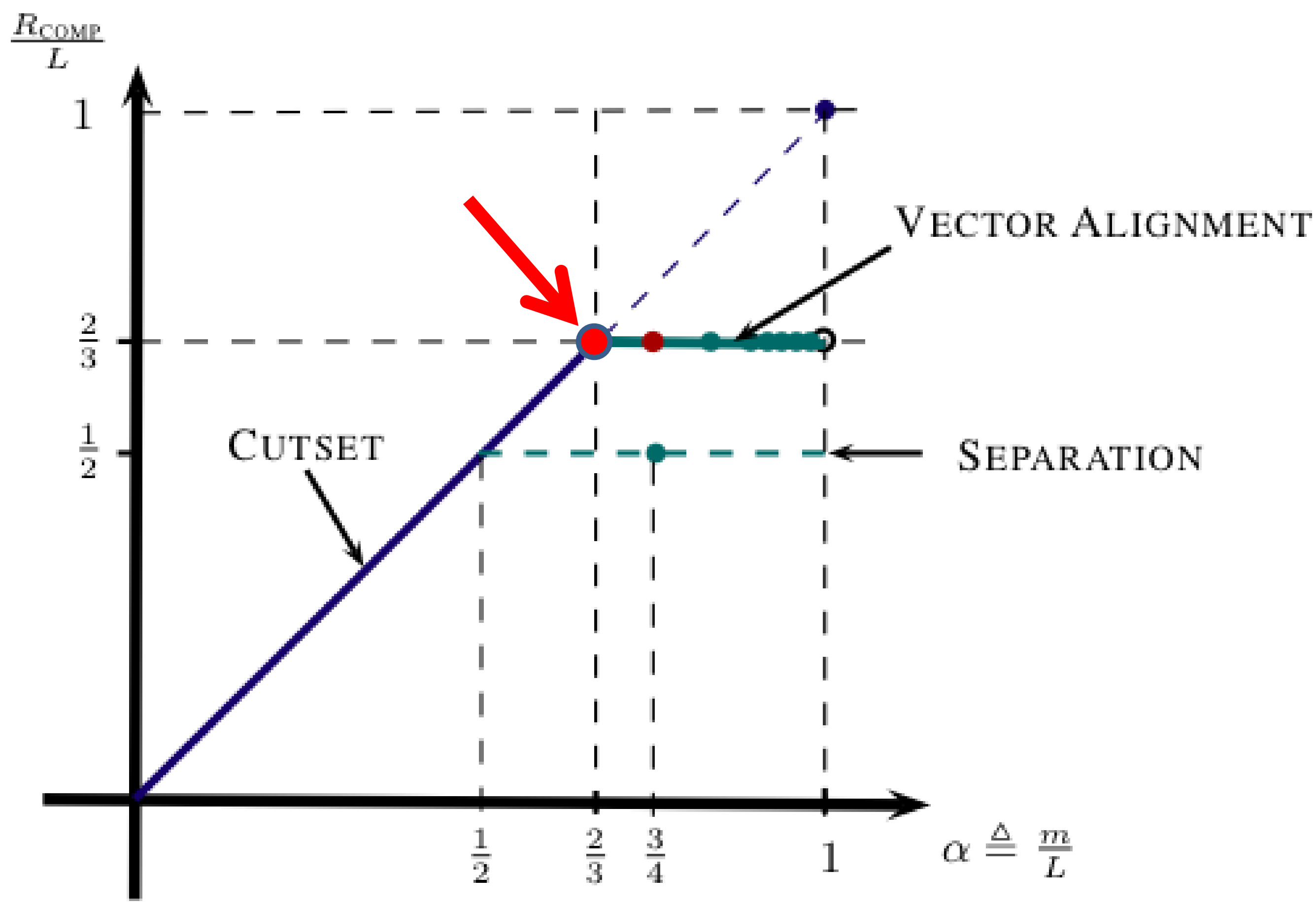
Countably Infinite Class of (m, L) Networks



Main Capacity Result



Main Capacity Result



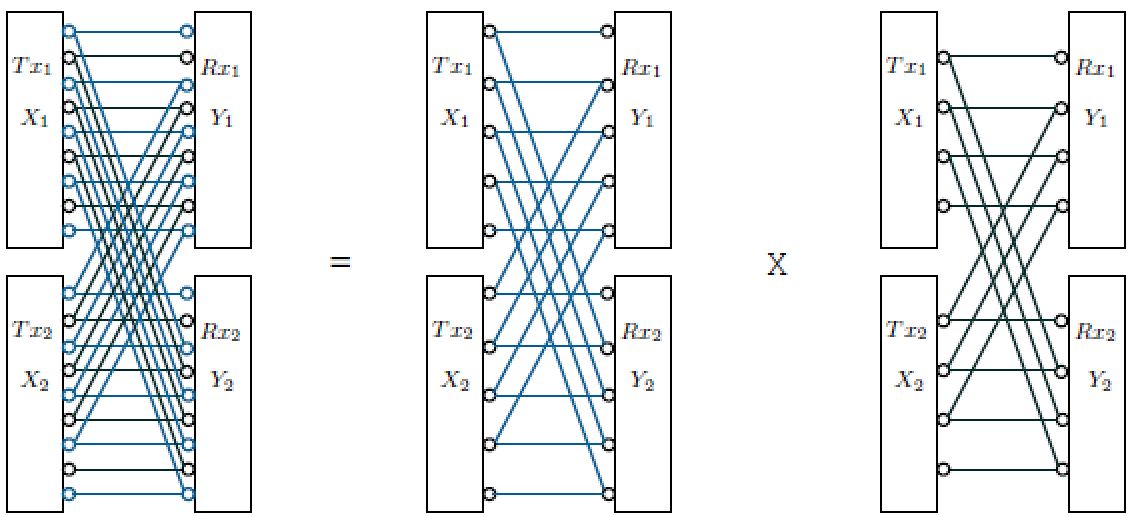
Network Decomposition

An (m, L) network decomposes as

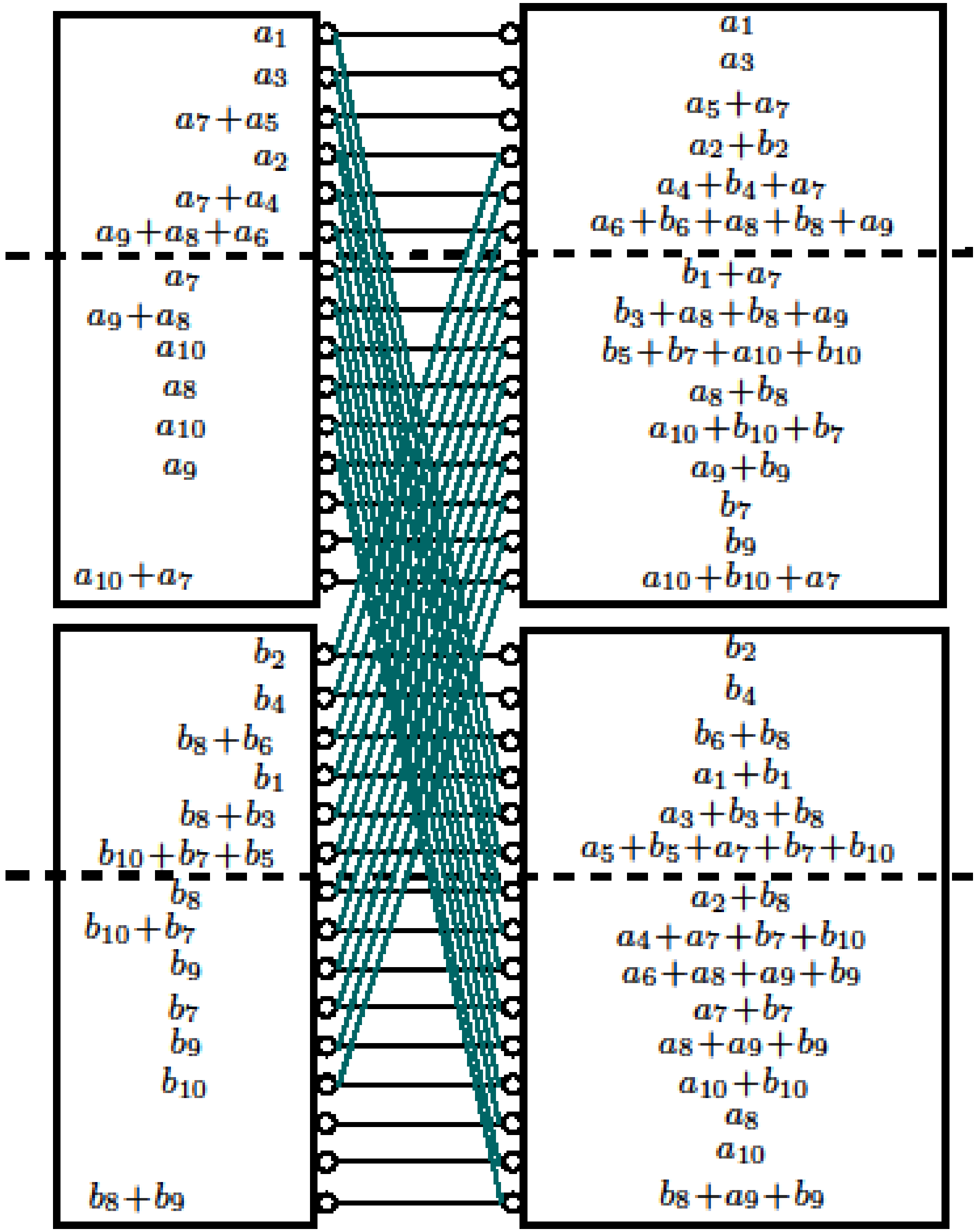
$(m, L) = (r, r+1)^{L-m-a} \times (r+1, r+2)^{a}$ where

$r = \left\lfloor \frac{m}{L-m} \right\rfloor$ and $a = m \mod (L-m)$.

Example: $(7, 9) = (3, 4) \times (4, 5)$.



Code for $(m, L) = (12, 15)$ Network



Material Presented From Following References

N.G., Suh, Gastpar, “Network Coding With Computation Alignment,” IEEE ITW Lausanne, Switzerland, 2012.

Suh, N.G., Gastpar, “Computation in Multicast Networks: Function Alignment and Converse Theorems,” Submission to IEEE IT Transactions, Reference: http://arxiv.org/abs/1209.3358, 2012.

N.G., Gastpar, “Reduced-Dimension Linear Transform Coding of Correlated Signals In Networks,” IEEE Transactions on Signal Processing, vol. 60, no. 6, pp. 3174-3187, June, 2012.

Thank You

Thank You

Enjoy the Fresh Strawberry and Raspberry Chocolate Cake!