Vibrational behavior of a soundbox in an atmosphere with a variable speed of sound

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This paper describes a semi-quantitative method, suitable for a student laboratory exercise that shows that the acoustic properties of the soundbox of a musical instrument depend on the sound speed of the atmosphere surrounding and filling the instrument. A gas tent was constructed and used to enclose instruments in helium, carbon dioxide and mixtures thereof, allowing the sound speed to be varied from 250 to 1000 m/s. Soundboard admittance data were taken using a guitar and a violin as examples. The data, expressed as contour plots, show clearly the qualitative relationship between air and wood modes, and the guitar data are compared with a simple mechanical model. Experimental details of the construction and operation of gas tent are given, with attention paid to safety issues. © 2012 Acoustical Society of America. [DOI: 10.1121/1.3677250]

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I. INTRODUCTION

Most string instruments have a soundbox that is essentially a wooden shell with a hole, or holes. The quality of sound production from such a shell is dominated by the interplay between the vibrational modes of the soundboard, and those of the air enclosed in the soundbox and in the necks of the hole(s). The air causes the lowest soundboard mode to be split into two, depending on whether the air is moving in phase, or out of phase, with the motion of the soundboard. An essential feature of this system is that the frequency interval between the two resonances has no deep minimum in the sound radiation. It is important that this interval be broad and that it should lie at the low-frequency end of the instrument’s range, where sound production would otherwise be inefficient. Depending on the relative contribution to the sound field of the soundboard and soundhole air movement, the two resonances are conventionally known as A0 and T1, with the former having the greater contribution from the soundhole.

This paper describes a semi-quantitative method, suitable for a student laboratory exercise that shows that the acoustic properties of the soundbox of a musical instrument depend on the sound speed of the atmosphere surrounding and filling the instrument. By examining soundboxes in an environment of changing sound speed one can separate out the contributions of “air modes” and “wood modes” to the radiated sound. A pure air mode would behave like a Helmholtzian and its frequency would be proportional to the local sound speed. A pure wood mode would have a much smaller dependence on sound speed and would vary only slightly due to the changing mass loading of the surrounding gas. By tracking the frequencies of these modes as a function of sound speed it is possible to identify directly which modes are predominantly “air” and which are predominantly “wood.”

A gas tent was constructed and used to enclose instruments in helium, carbon dioxide and mixtures thereof, allowing the sound speed to be varied from 250 to 1000 m/s. Soundboard admittance data were obtained remotely using a shaker and an accelerometer. Two familiar and acoustically well-studied instruments were used as examples: a guitar and a violin (both reasonable student instruments, but in no way exceptional). Their vibrational modes were established independently by standard modal analysis, and in the case of the guitar, a simple electrical circuit analog model. Thus, a basis was provided for comparison with the sound speed experiment.

Section II of this paper describes various methods of examining air and wood modes. Details of the construction and operation of gas tent, and the manner in which the measurements were taken are discussed in Sec. III, with Secs. III A and III B being devoted to admittance data taken with different sound speeds and the modal analysis, respectively. The results Sec. IV is divided into Sec. IV A for the guitar data and Sec. IV B for those of the violin.

II. DISTINGUISHING AIR AND WOOD MODES

Consider a typical soundbox as shown in Fig. 1. An experimental determination of the contributions of wood and air to these low frequency resonances can be made in a number of ways. One is free to vary in any combination the effective mass of the soundboard m, the mass of the air (gas) moving through the soundholes μ, its bulk modulus B, density ρ0, and sound speed c. Some of the ways in which this has been done experimentally are listed below.

1. Packing the soundbox in sand or plaster of paris immobilizes the wood and allows an examination of the “pure” air-modes, of both the Helmholtzian and pipe variety.

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This technique has been used both to study air-modes (e.g., for the harp in Ref. 2) and also to immobilize just part of an instrument (e.g., the back of a guitar, Ref. 3) to facilitate analysis.

2. The addition of a “collar” or “chimney” to the soundhole increases the mass of the air piston $\mu$ and reduces the frequency of the air-like modes. Generally used together with method 3 (below), this technique has been used on the guitar$^4$ and harp.$^5$

3. The addition of small masses to the soundboard increases its effective mass $m$ at antinodal positions and reduces the frequency of the wood-like modes. The relative size of frequency shifts caused by increasing the mass of the oscillating air and the soundboard alternately shows which mode behaves more like an air-mode and which more like a wood mode.$^4,5$

4. Adding holes in the ribs of the instrument decreases the effective density $\rho_0$ of the air while maintaining the bulk modulus $B$ and at the same time keeping the sound speed constant. Used once (to our knowledge) and to great effect in Hutchins’ “Le Gruyère” violin,$^6$ and analyzed by Shaw$^7$ and Weinreich,$^8$ the quality of sound radiation was measured as a function of the changing air resonance frequencies.

5. Removal of the air by placing the instrument in a vacuum chamber reduces $B$ and $\rho_0$ while keeping $c$ constant, thus the system reduces to one of simple wood modes only. The evacuation technique has been used by David to study the radiated sound of a guitar,$^9$ but not to examine air-wood coupling.

6. Raising or lowering the temperature varies the sound speed $c$, but has not been used, presumably because of the danger to the instrument.

7. Changing the gas varies $\rho_0$ (and to a small extent $B$, via the adiabatic index $\gamma$), thus varying $c$. This method has been used to study air modes of the violin$^{10}$ and guitar,$^{11}$ and is the technique discussed here. Placing the instrument in an atmosphere of another gas is the most straightforward way to vary the atmospheric sound speed. In the case of a pure wood mode, little variation is to be expected with atmospheric sound speed. In contrast, any pure air mode would have a frequency that is linearly dependent on sound speed. Hence by observing how a physical mode changes frequency with sound speed it can be seen how much of the motion of the mode is due to air and how much is due to wood. McLennan$^{10}$ studied a violin in helium, air and carbon dioxide, tapping the soundboard at the bridge and measuring the sound inside the instrument. The frequencies of major resonances, particularly the A0 and cavity modes, were analyzed as a function of the gas mixture. Ezcurra et al.$^{11}$ analyzed the modes of vibration of a guitar in pure helium, air and krypton and were able to quantify the contributions of air and wood for these three cases. In particular they showed that both the A0 and T1 modes rose in frequency with a rise in sound speed, but the A0 changed more rapidly.

Consider the frequency of a Helmholtz resonance of frequency $f_H$ in a rigid cavity with a hole:

$$f_H = \frac{c}{2\pi} \sqrt{\frac{A}{V + 2dH}} \approx \frac{A}{V},$$

where $c$ is the sound speed, $V$ is the volume, $A$ is the area of hole, $l$ is the thickness of soundboard at the hole, and $dl$ is the hole thickness correction.$^{12}$ The same dependence on sound speed can be expected for any cavity mode with the soundbox immobilized. In the admittance measurement reported here, a mode like the Helmholtz resonance tends to show up as an antiresonance, i.e., a minimum in the admittance, precisely because the soundboard moves the least at this frequency.

III. THE MEASUREMENTS

A. Changing the sound speed

1. The gas tent

A gas tent was constructed, 1.3 m high with a 0.5 m by 0.5 m base, large enough to contain a string instrument up to the size of a small harp, in which the mechanical admittance of any structure may be measured. The atmosphere inside the tent was helium, air, carbon dioxide or a mixture thereof. In this manner any sound speed between about 250 m/s and 1000 m/s could be obtained, and the value at any instant was measured using the pipe resonances of a meter-long tube. Gas mixing was assisted by the use a small fan inside the tent. The admittance measurements were made with a shaker and an accelerometer. We took care to establish that these measurements were not significantly affected by the tent or its contents.

The frame of the gas tent was made from 2-inch diameter PVC pipe, glued together with standard L-bend and T-junction fittings. The top and the bottom were squares 0.5 m on each side. These two squares were joined by four pipes 1.3 m long to form the vertical sides. The gas was introduced into the top square which had several small holes drilled on the inside to allow gas to enter the body of the tent. The top square was internally isolated from the rest of the frame so gas could not flow out anywhere else. In the bottom square were two holes, one for cables that was sealed, and one connected to a pump that allowed gas to be removed and exhausted outside the building. The frame was covered with two layers of 0.15 mm thickness polyethylene sheet; one layer allowed helium to diffuse out too quickly. One face of the frame was formed into a “door” by covering it with double polyethylene sheet taped at the edges with 3-inch wide packing tape. This door was easily removed to allow access to the inside. The completed tent is shown in Fig. 2.
In one corner of the tent was placed a cardboard tube, 94 cm long and 15 cm diameter, the purpose of which was to make a sound speed measurement. This wide tube was chosen both to allow swift mixing of the gas with the rest of the tent, and also to avoid nonlinearities associated with narrow tubes. In one end of the tube was placed a small loudspeaker, and in the center a microphone. The speaker was fed a 4-s linear chirp from 50 to 1500 Hz, and the microphone output was analyzed for resonant frequencies.

The tent was filled by letting the gas into the top of the tent while pumping from the bottom. The pump was controlled by a Variac that was set to keep the flexible walls of the tent from straining in or out. The sound speed was monitored continuously, with the fan being used occasionally to hasten the mixing of the gas. Once the sound speed had reached its maximum or minimum value, depending on whether the gas used was helium or carbon dioxide, the gas flow and pump were switched off. The filling took about an hour.

Helium diffused out of the tent with a time constant of about an hour; this was still too fast, so helium was bled in during the data taking to slow the rate of change and ensure good mixing inside the tent and instrument. The optimum procedure was established by taking measurements at various rates of change. On the other hand, the carbon dioxide diffused out slowly, and little change was observed for several hours after filling. As a result, the operation was generally hastened by the use of a controlled leak and the pump.

The diffusion constants for the gas mixtures in use are \( D \approx 0.16 \, \text{cm}^2/\text{s} \) for CO\(_2\)-air and 0.6–0.7 \( \text{cm}^2/\text{s} \) for He-air (depending on the concentration ratio). Given an elapsed time, \( t \), the diffusion length \( L_D = 2\sqrt{Dt} \) can be calculated. A typical time between measurements was about 20 min during the helium fill, and about an hour during the CO\(_2\) fill, for which in both cases \( L_D \approx 50 \, \text{cm} \). The bulk of the gas in the tent was stirred by a fan, so this diffusion length was adequately long for the size of a guitar or violin soundbox, where the gas should be assumed conservatively to be stagnant and mixing only by diffusion. In contrast, the cardboard tube used to measure the sound speed was wide and oriented in the direction of the gas circulation and therefore expected to be fully mixed inside.

Several hours in a zero humidity environment can be detrimental to a wooden soundbox, so in every case, a water tray was placed on the floor of the tent to maintain a safe humidity level. In early runs, a humidity meter was used to ensure that the water tray was effective. The relative humidity dropped to about 35\% briefly during the gas fill, but thereafter stayed between 60\% and 70\%; these values are typical of those found in Vancouver buildings.

It should be noted that this type of experiment was, and should always be, performed in a large room with open windows. An oxygen meter should be kept on the laboratory bench. The tent should be thoroughly emptied of CO\(_2\) or helium and vented outside before opening the front flap. The work should not be done alone. Although the total tent volume (0.325 \( \text{m}^3 \)) was tiny compared to that of the room, a sudden rupture or careless opening could cause a dangerous local concentration of asphyxiant gas.

2. Admittance measurements

The admittance of the soundboard of each instrument was measured at the most appropriate point, i.e., the places where it is excited into vibration by the strings. For the guitar this point was the center of the bridge; for the violin, the foot of the bridge on the G-string side. The instruments were hung from the ceiling of the tent. A small Brüel and Kjaer\(^1\) 0.6 g accelerometer was attached with wax and the stinger of a Brüel and Kjaer shaker was attached to the accelerometer, also with wax. Care was taken with the positioning of the shaker so that the stinger was neither in compression nor tension. The shaker was driven by a 4-s linear chirp 50–1500 Hz. Figure 2 shows the shaker and stinger attached to a guitar visible through one sheet of polyethylene in an early version of the tent. The force was calibrated by making the same measurement, in air, with an impact hammer.\(^1\) In the case of the guitar, the modes were observed at the same frequencies by both methods so calibration was straightforward. For the violin, small shifts were seen in modal frequencies as a result of the shaker mass, but the modes were still identifiable. Each admittance measurement was correlated with a sound speed measurement, and the data were used to form contour plots. Data were taken at different rates of change and it was established that for the times given in Sec. III A 1 or slower, the results were consistent with each other.

B. Modal Analysis

In order to confirm, for the purposes of this paper, the nature of the modes seen in the admittance measurements, the instruments were subjected to standard modal analysis.\(^1\) This standard, if specialist, procedure is not crucial to the basic gas tent experiment, and it will not be described here in any detail.
IV. RESULTS

A. Guitar

1. Admittance contour plot

The admittance contour plot for the guitar is shown in Fig. 3. The horizontal axis denotes, strictly speaking, the speed of sound in the tube, rather than the instrument, but the two are expected to be similar. The most prominent features at low frequency are the bands of high admittance at about 100 and 200 Hz and the line of low admittance between the two. These are, respectively, the A0 and T1 resonances, and the Helmholtz antiresonance. The A0 and T1 are formed by the coupling of the lowest top plate resonance with the Helmholtz resonance of the enclosed air of the soundbox. It is clear from this plot that the lower of the two resonances is the A0, because it changes much more rapidly with sound speed than the upper resonance, and thus it has the larger share of the air motion.

At higher frequencies there is less coupling between air and wood. Higher wood resonances are seen as horizontal lines, with weaker diagonal air resonances crossing these occasionally. Rossing et al.¹⁹ report the first cavity mode, the A1 resonance, in a Martin D-28 guitar with an immobilized soundbox to be at 383 Hz. Ezcurra et al.¹¹ see the same resonance in a craftsman-made instrument at 418 Hz. A diagonal structure can be seen in the top left of Fig. 3 crossing 340 m/s at about 400 Hz, and this is most likely a result of the A1 mode. As the A1 frequency is a function only of the soundbox geometry, which changes little between instruments, it is not surprising that it appears in this soundbox at a similar frequency to those cited by other authors.

The identification of the resonances seen in the modal analysis is shown on Fig. 3. Apart from the A0 and T1, one can see the T1' mode (which is related to the T1, the two modes being split by coupling to the guitar back), bending modes whose frequency is almost independent of sound speed, and higher modes of the front and back plates. The last are labeled (i, j; k, l), where the numbers refer to the number of nodal lines in the transverse and longitudinal directions, for the front and back plates, respectively.³

2. A simple mechanical model of the guitar

In the case of a guitar with its well-separated resonances, it is possible to calculate the soundboard admittance with a mechanical model of three masses coupled with springs, or a simple electrical circuit analog.³,¹⁶,¹⁷ Whilst not a detailed representation of the vibro-acoustics of the guitar, it does reproduce the broad low frequency features and can easily be modified to include the change of gas. It is, therefore, a useful pedagogical tool to help understand the distinction between the A0 and T1 modes.

Model parameters independently measurable were the volume of soundbox, 13 L, and the radius of the soundhole, 43.5 mm. Values found to give qualitative agreement with our measurements were \( m = 0.11 \) kg, \( k = 150 \) kN/m for the top plate (area 0.060 m²), and \( m' = 0.15 \) kg, \( k' = 150 \) kN/m for the back plate (area 0.088 m²). The effective mass of air in the soundhole was found by the standard procedure (Eq. (1)). The variation of these frequencies with gas parameters \( c \) and \( \rho \) was calculated and the results shown in Fig. 4. The sound speed of gas mixtures was calculated from the molar averages of the densities and the specific heats. As can be seen by comparison with the data in Fig. 3, the trends in the resonant frequencies are reproduced, if not the detailed numerical values. Of course, the model is an oversimplification of a physical guitar, and there are many more than three significant resonances, all of which interact with each other.³

B. Violin

The admittance contour plot for the violin is shown in Fig. 5. Once again the horizontal axis denotes, strictly speaking, the speed of sound in the tube. The most striking feature

FIG. 3. Driving point admittance \( Y \) (in s/kg) of the guitar at the bridge, as a function of sound speed. The vertical axis denotes frequency; the horizontal axis denotes the sound speed in the tube in the gas tent. Bright features have high admittance; dark areas have low admittance. The bright line crossing 100 Hz at 340 m/s is the A0 resonance; the 200 Hz bright line is the T1. The fact that the lower resonance has the stronger dependence on sound speed indicates that it is the A0. Higher modes of the front and back plates are labeled \((i, j; k, l)\), where the numbers refer to the number of nodal lines in the transverse and longitudinal directions, for the front and back plates, respectively.³ The diagonal structure crossing 400 Hz at 340 m/s is probably the A1 cavity resonance. Narrow horizontal lines are bending modes in the guitar and neck that do not significantly participate in sound radiation.

FIG. 4. Frequencies of the three modes (A0, T1, T1') in the three-mass model of the guitar, plotted against sound speed, and superimposed on the admittance contour plot of Fig. 3. In addition the expected trajectory of the Helmholtz antiresonance (H) is shown.
is a diagonal line apparently pointing back to the origin (c = 0, f = 0) and crossing 340 m/s at 295 Hz. This is the Helmholtz (anti-)resonance that appears as a dip in the soundboard admittance of all great violins at about 300 Hz. As a function largely of soundbox volume and f-hole area, there is little variation in this frequency from instrument to instrument. Other significant frequencies are much more variable. In this case, the A0 appears at 280 Hz and the T1 resonance shows up as a more-or-less horizontal band at about 500 Hz (a little below the 519 Hz indicated by the 507, DK-2850 Nærum, Denmark.

The diagonal dark line is the Helmholtz antiresonance. The notation of the identified modes follows Moral; the most efficient radiators are the A0 and T1 modes, while the C (corpus) and N (neck) modes are less so.

V. CONCLUSION

Measuring the vibrational properties of the soundboxes of string instruments by varying the sound speed provides another means of studying the air and wood modes of the soundbox. Data taken while changing the gas mixture surrounding and filling the instrument can be displayed in a visually appealing way that makes a clear distinction between the different mode types. The experimental technique is amenable to use in an undergraduate acoustics laboratory.

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