Exchanging Information with the Stars

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The goal

- Exchanging information with other solar systems would be an exciting voyage
- The capabilities and limitations of our Universe to support such exchanges is little understood
- This work is a first step toward such understanding

Some challenges

- No experimentation
  - Relevant astronomical observations
- No coordination
Implicit coordination

Design guidance based on:

▶ Simplicity: Occam’s razor
▶ Fundamental limits and resulting optimization
▶ Where physical impairments are least controlling
▶ Assumptions about capabilities and resources
▶ Awareness of motivations and incentives

Some distinctions

▶ Attractor beacon vs. information-bearing signal
▶ Discovery vs. ongoing communication

This talk focuses on:

▶ Radio frequencies
▶ Design of an information-bearing signal
▶ Receiver design for discovery of that signal

Impairments addressed in this talk

▶ White noise
▶ Radio-frequency interference
▶ Dispersion in the ionized interstellar medium (IISM)

Complex-valued baseband equivalent signal
Digital modulation alternatives

Focus on the complex baseband signal:

- **Data symbols** \( \{ B_k \} \)
- **Amplitude modulation:**
  \[
  \{ B_k \cdot h(t - k T_s), -\infty < k < \infty \}
  \]
- **Orthogonal signaling:**
  \[
  \{ h_{B_k}(t - k T_s), -\infty < k < \infty \}
  \]

Discovery options

- **Multiple-symbol**: Make additional assumptions about data symbol alphabet
- **Symbol-by-symbol**: Single symbol waveform \( h(t) \) multiplied by some unknown amplitude and phase

Here we pursue the symbol-by-symbol option:

- Applies to all modulation alternatives
- Potentially forgos signal energy

Time-frequency support for \( h(t) \)

Transmitter:

- What should \( W \) and \( T \) be?
- What other properties should \( h(t) \) have?

Receiver:

- How advantageous is it to know more about \( h(t) \)?
- How does the receiver infer this knowledge?

Received signal impairments

Temporarily consider only:

- **White Gaussian noise**
- **Radio interference** in the vicinity of the receiver

Optimization infers specific and credible properties for \( W \), \( T \), and \( h(t) \)
Two orthonormal bases

An orthonormal basis renders the reception finite-dimensional:

- **Fourier series** (time-limited signal)
- **Sampling theorem** (bandlimited signal)

Isotropic noise

- Matched filter looks in the signal direction
- Sensitivity depends on $\mathcal{E}_s$ and $\sigma^2$...
- ...and not $W$, $T$, and the “shape” of $h(t)$

Finite-dimensional representation of $h(t)$

Regardless of basis:
- Noise is completely random and isotropic

Choice of basis:
- Transmitter and receiver must assume the same basis
- We choose the Fourier series

Dimensionality of basis:
- Degrees of freedom (DOF) is $K = W \cdot T$

Radio-frequency interference

- How to best deal with interference depends on its characteristics

Narrowband case:
- Want signal energy uniformly distributed over $0 \leq f \leq W$
- Want $W$ large and $T$ small
Interference

**Broadband interferer:**

- Want signal energy uniformly distributed over $0 \leq t \leq T$
- Want $T$ large and $W$ small

Ideal signal design for interference

To counter interference, the signal should be isotropic:

- Statistically completely random
- In-band interference energy is reduced by $1/K$ after matched filtering
- Spread spectrum: Want $K = W \cdot T$ large
- Current and past searches will likely miss this signal

Pseudo-random signal based on $\pi$

Binary expansion of $\pi$ is history’s most studied pseudo-random generator:

Some environmental factors

**Time-invariant**
- Plasma dispersion
- Scattering

**Time-varying**
- Doppler
- Turbulence
- Scintillation (fading)
Bandwidth stress test of the ISM

High data rate. $W \cdot T \approx 1$ and $1/T$ large
Spread spectrum. $W \cdot T >> 1$

We choose spread spectrum:
- Suppresses interference
- Usually less affected by multipath
- Discovery is easier
- ISM bandwidth is “free”

Plasma dispersion

The ISM is conductive due to ionization in interstellar gas clouds:

**Homogeneous refractive index:**

$$n = \left(1 - \left(\frac{f_p}{f}\right)^2\right)^{-1/2}$$

**Group delay:**

$$\tau(f) = \frac{D \cdot DM}{f^2}$$

- $f =$ frequency in Hz
- $D \approx 4.15 \times 10^{15}$ Hz$^2$ pc$^{-1}$ cm$^3$ s
- $DM =$ column density of electrons, ranges between $\approx 1$ and $\approx 1000$ cm$^{-3}$ pc

Delay spread

The delay spread $\tau_{\text{max}} =$ range of group delays across $f_c \leq f \leq f_c + W$:

$$\tau_{\text{max}} = \tau(f_c) - \tau(f_c + W)$$

Delay spread vs $f_c$

Dispersion favors large $f_c$: $\tau_{\text{max}} \downarrow$ rapidly as $f_c \uparrow$

$W = 1$ MHz, $DM = 1, 10, 100, 1000$
Relation of group delay and phase

Frequency response of propagation:

\[ F(f) = |F(f)| \cdot e^{i\phi(f)} \]

Monochromatic phase shift:

\[ 2\pi \cdot \tau(f) = -\frac{d\phi(f)}{df} \]

Typical case

Delay changes linearly and phase quadratically

\[ f_c = 1 \text{ GHz}, \ W = 1 \text{ MHz}, \ DM = 100 \]

Phase after wrapping

\[ f = \frac{m}{T}, \ T = 2 \text{ msec}, \ 0 \leq m < 2000 \]

Impulse response energy is spread uniformly over \( 0 \leq t \leq \tau_{\text{max}} \) but phase is chaotic

\[ \text{DFT}^{-1}\{e^{i\phi_m}\} \text{ and } \tau_{\text{max}} \approx 0.8 \text{ msec} \]
Fourier-series representation of $h(t)$

Fourier-series basis for an isolated pulse $h(t)$ is a natural for characterizing dispersion:

$$h(t) = \sqrt{E_s} \cdot w(t) \cdot \frac{1}{\sqrt{K}} \sum_{m=0}^{K-1} c_m \cdot e^{2\pi mt/T}$$

Effect of delay spread on one component of $h(t)$

Assuming $\phi(f) \approx$ linear for small $|f - f_0|

\[
\begin{array}{c}
w(t) e^{i2\pi f_0 t} \\
\rightarrow |F(f)| e^{i\phi(f)} \\
\rightarrow e^{i\phi(f_0)} w(t - \tau(f_0)) e^{i2\pi f_0 t}
\end{array}
\]

- Effect of group delay on $w(t)$ can be ignored if $\tau_{\text{max}} << T$
- Argues in favor of choosing $T >> \tau_{\text{max}}$

Filter bank receiver processing

Filter bank: One channel (out of $K$)

$$Y(t) \rightarrow \otimes \rightarrow w^*(-t) \rightarrow \text{Sample } t = 0 \rightarrow Y_m$$

\[
Y_m = \sqrt{\frac{E_s}{K}} \cdot c_m \cdot e^{i\phi_m} + N_m \quad 0 \leq m < K
\]

Receiver de-spreading

Spread the interference without affecting the noise statistics:

$$Y_m \rightarrow \otimes \rightarrow P_m$$

\[
P_m = \left(\sqrt{\frac{E_s}{K}} + O_m\right) \cdot e^{i\phi_m} \\
E |O_m|^2 = \sigma^2
\]
Performance metric

What increase in $\varepsilon_s$ maintains fixed $P_{FA}$ and $P_D$?

Asymptote for large $K$:

$$E_s \sim f(K) \text{ if } E_s \to \alpha \cdot f(K) \text{ as } K \to \infty$$

Central limit theorem and law of large numbers apply

Values we encounter

<table>
<thead>
<tr>
<th>Algorithm</th>
<th>$E_s \sim ??$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incoherent matched filter</td>
<td>$1$</td>
</tr>
<tr>
<td>Maximum likelihood</td>
<td>$\sqrt{\log K}$</td>
</tr>
<tr>
<td>Energy estimation</td>
<td>$\sqrt{K}$</td>
</tr>
<tr>
<td>Dispersion estimation</td>
<td>$K$</td>
</tr>
</tbody>
</table>

Energy penalty

- $E_s \sim \{1, \sqrt{\log K}, \sqrt{K}, K\}$
- At $K = 10^6$, $E_s \sim \{1, 3.7, 10^3, 10^6\}$

Energy vs power

<table>
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<tr>
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<th>$E_s$</th>
<th>$P_s$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Incoherent matched filter</td>
<td>$1$</td>
<td>$\frac{1}{T}$</td>
</tr>
<tr>
<td>Maximum likelihood</td>
<td>$\sqrt{\log(W \cdot T)}$</td>
<td>$\frac{\sqrt{\log(W \cdot T)}}{T}$</td>
</tr>
<tr>
<td>Energy estimation</td>
<td>$\sqrt{W \cdot T}$</td>
<td>$\sqrt{\frac{W}{T}}$</td>
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<td>$W \cdot T$</td>
<td>$W$</td>
</tr>
</tbody>
</table>
Known $\tau_{\text{max}}$: incoherent matched filter

Assuming $\tau_{\text{max}}$ (hence $\phi_m$) is supplied by a genie:

\[ P_m \xrightarrow{\text{Phase equalizer}} e^{-i\phi_m} \xrightarrow{\text{Matched filter}} \frac{1}{\sqrt{K}} \sum_{m=0}^{K-1} \xrightarrow{\text{Incoherent carrier phase}} |\cdot| \xrightarrow{\text{Incoherent matched filter}} Q \]

\[ \mathcal{E}_s \sim 1 \]

Detection based on energy estimation

Estimating signal energy does not require knowledge of $\tau_{\text{max}}$ or $\{c_m\}$:

\[ Y_m \text{ or } P_m \xrightarrow{|\cdot|^2} \sqrt{\sum_{m=0}^{K-1}} \xrightarrow{\text{Q}} Q \]

\[ \mathcal{E}_s \sim \sqrt{K} \]
**Partial equalization for restricted delay spread**

- A priori knowledge of group delay
- Equalization for minimum DM
- Smaller delay spread

**Maximum delay spread**

- For specific $f_c$, $W$, and LOS the delay spread is bounded by $\tau_{\text{max}} \leq \Omega$
- Knowing $\Omega$, we search over $T > \Omega$
- The duration of the impulse response $\Omega < T$

**Restricted-delay spread energy estimation**

- $P_m$ → Min DM delay equalizer → Impulse response $\text{DFT}^{-1}$ → Partial energy $\sum_{k=0}^{L} |\cdot|^2$ → $Q$

\[ \mathcal{E}_s \sim \sqrt{\Delta r \cdot W} \text{ becomes independent of } T \]

**Maximum likelihood**

- Find $L$ orthonormal basis vectors that represent $e^{i\phi_m}$ for any $0 \leq \tau_{\text{max}} \leq \Omega < T$

\[ L \approx \frac{1}{2} \cdot \frac{\Omega}{T} \cdot K \]

- Find that basis vector most likely to represent filter bank output:
  - $Q_n = \text{IMF}$ for $n$'th basis
  - $\max_n Q_n = \text{threshold input}$
- Resulting energy penalty is small:

\[ \mathcal{E}_s \sim \sqrt{\log L} \]
Nonlinear DOF reduction

The first difference of phase is a slowly varying function of \( m \):

\[
\Delta \phi_m = \phi_{m+1} - \phi_m \approx -\frac{2\pi}{T} \cdot \tau \left( \frac{m}{T} \right)
\]

Orthonormal basis for \( e^{i\Delta \phi_m} \)

- \( \{ e^{i\Delta \phi_m}, 0 \leq m < K \} \) is always less than one period of a complex exponential

- \( L = 5 \) orthonormal basis functions

- DOF \( L = 2 \) suffices for most purposes

Nonlinear DOF reduction

A noisy estimate of \( e^{i\Delta \phi_m} \) can be formed from the filter bank output:

\[
P_{m+1}P_m^* = \left( \sqrt{\frac{\mathcal{E}_s}{K}} + O_{m+1} \right) \left( \sqrt{\frac{\mathcal{E}_s}{K}} + O_m \right)^* \cdot e^{i\Delta \phi_m}
\]

- Use the ML approach for \( P_{m+1}P_m^* \), but with only \( L = 2 \) basis vectors
- Like all autocorrelation algorithms:
  - \( \mathcal{E}_s \sim \sqrt{\mathcal{R}} \), same as energy estimator
  - Results from the noise-on-noise \( O_{m+1}O_m^* \) term

Estimation of dispersion

A noisy estimate of \( \Delta \phi_m \) can be formed from the filter bank output:

\[
\arg \left( P_{m+1}P_m^* \right) = (\Delta \phi_{m+1} + \Theta_{m+1} - \Theta_m) \mod 2\pi
\]

- Slope of \( \Delta \phi_m \) vs \( m \) is proportional to \( \tau_{\text{max}} \)
- \( \Theta_m \to \) uniform distribution on \([0, 2\pi]\) unless \( \mathcal{E}_s \sim K \)
- \( \mod 2\pi \) nonlinearity is the killer
Phase estimation and unwrapping

\[ \text{arg} (P_{m+1} P_m^*) \]

\[ \frac{\varepsilon_s}{\sigma^2} = \alpha \cdot K \]

\[ K = 1000 \text{ (30 dB)} \]

\[ \alpha = 10, 3, \text{ and 0 dB} \]

Conclusions regarding dispersion

- Direct estimate of \( \tau_{\text{max}} \) is too noisy
- Maximum likelihood detection requires a modest penalty in \( \varepsilon_s \) but is computationally expensive
- Energy estimation for impulse response \( 0 \leq t \leq \tau_{\text{max}} \) is low complexity but requires larger increase in \( \varepsilon_s \)
- Search parameters:
  - Large \( f_c \) to reduce \( \varepsilon_s \) penalty and computational burden
  - Search over \( T > \tau_{\text{max}} \), but \( T \) not so large that time-varying effects come into play
  - A search over \( W \) is not necessary

Summary

- In white Gaussian noise, detector should use a matched filter
- In radio-frequency interference, signal optimally appears statistically like
  - Bandlimited and time-limited white Gaussian noise
  - Large \( W \cdot T \)
- ISM bandwidth stress test demonstrates tradeoff between computational burden and
  - Carrier frequency \( f_c \)
  - Prior knowledge of dispersion measure DM
  - Received signal energy penalty
  - Interference rejection
- Scattering and fading under study

Takeaways

- Optimization provides implicit design coordination
- Propagation impairments constrain search parameters
- The more a priori knowledge of the signal, the more sensitive its detection
- Communication engineering is immediately relevant to SETI
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Further information

My homepage:

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