Exchanging Information with the Stars

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Overall goal

- Exchanging information with civilizations living in other solar systems would be an exciting voyage
- The capabilities and limitations of our Universe to support such exchanges is little understood
- This work is a first step toward such understanding

Some challenges

- No experimentation
  - Relevant astronomical observations
- No coordination

Implicit coordination

Design guidance based on:

- Simplicity: Occam's razor
- Fundamental limits and resulting optimization
- Where physical impairments are least controlling
- Assumptions about capabilities and resources
- Awareness of motivations and incentives

Some relevant distinctions

- Attractor beacon vs. information-bearing signal
- Discovery vs. ongoing communication

This talk focuses on:

- Radio frequencies
- Design of an information-bearing signal
- Receiver design for discovery of that signal

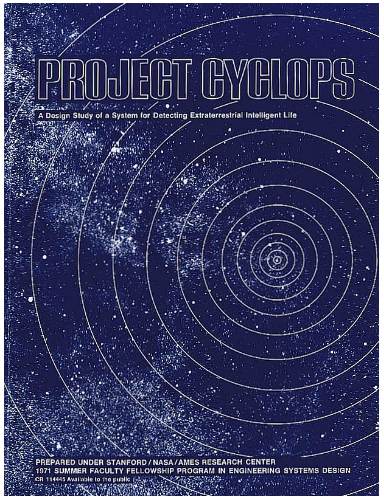
Immediate application

Allen Telescope Array (ATA), Hat Creek, California, is devoted to SETI observations

We seek to:

- Generalize the class of target signals
- Take advantage of advancing technology

CYCLOPS (1970)



CYCLOPS PEOPLE

Co-Directors:

Bernard M. Oliver	Stanford University (Summer appointment)
John Billingham	Ames Research Center, NASA

System Design and Advisory Group:

James Adams	Stanford University
Edwin L. Duckworth	San Francisco City College
Charles L. Seeger	New Mexico State University
George Swenson	University of Illinois

Antenna Structures Group:

Lawrence S. Hill	California State College L.A.
John Minor	New Mexico State University
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Harvey B. Sharfstein	San Jose State College
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Receiver Group:

Ward J. Helms	University of Washington
William Hord	Southern Illinois University
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C. Reed Predmore	Rice University

Transmission and Control Group:

Marvin Siegel	Michigan State University
Jon A. Soper	Michigan Technological University
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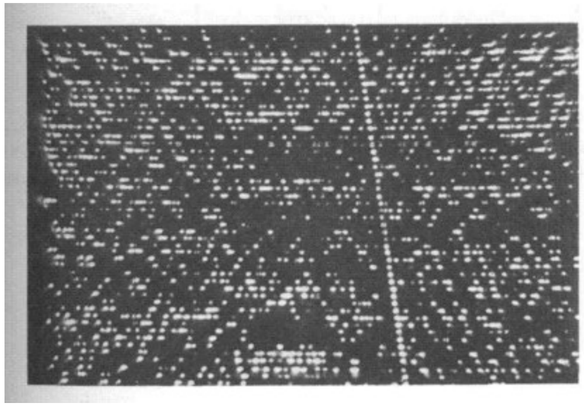
Signal Processing Group:

Jonnie B. Bednar	University of Tulsa
Douglas B. Brumm	Michigan Technological University
James H. Cook	Cleveland State University
Robert S. Dixon	Ohio State University
Johnson Luh	Purdue University
Francis Yu	Wayne State University

The Cyclops beacon signature

A spectrogram of a narrowband signal in noise with changing Doppler shift:

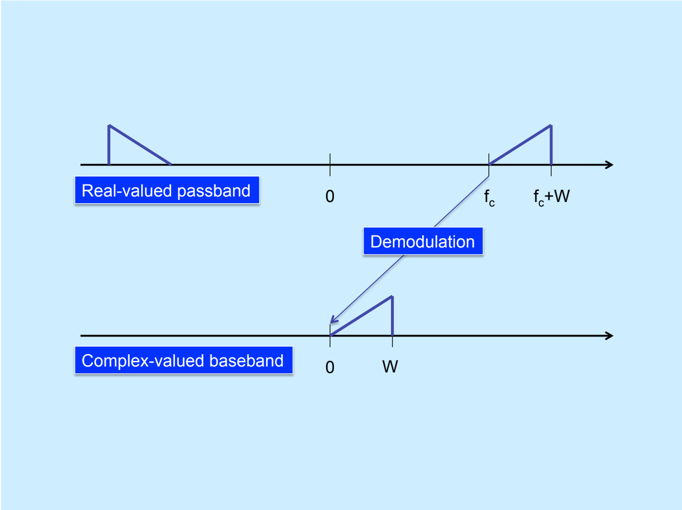


This talk

Implicit coordination between transmitter and receiver taking into account:

- White noise
- Radio-frequency interference
- Dispersion in the ionized interstellar medium (IISM)

Complex-valued baseband equivalent signal



Digital modulation alternatives

Complex-valued baseband signal:

- Data symbols $\{B_k\}$
- Amplitude modulation:

$$\{\, B_k \cdot h(t - k\,T_s),\ -\infty < k < \infty \,\}$$

- Orthogonal signaling:

$$\{\, h_{B_k}(t - k\,T_s),\ -\infty < k < \infty \,\}$$

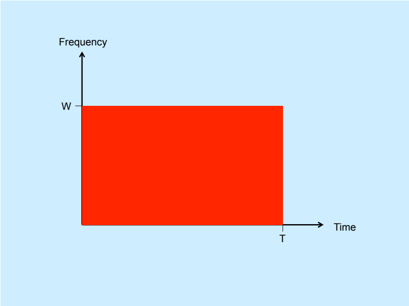
Discovery options

- Multiple-symbol: Make additional assumptions about data symbol alphabet
- Symbol-by-symbol: Single symbol waveform $h(t)$ multiplied by some unknown amplitude and phase

Here we pursue the symbol-by-symbol option:

- Applies to all modulation alternatives
- Potentially forgos signal energy

Time-frequency support for $h(t)$



Transmitter:

- What should W and T be?
- What other properties should $h(t)$ have?

Receiver:

- How advantageous is it to know more about $h(t)$?
- How does the receiver infer this knowledge?

Received signal impairments

Temporarily consider only:

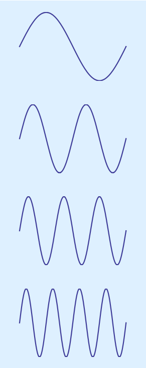
- White Gaussian noise
- Radio interference in the vicinity of the receiver

Optimization infers specific and credible properties for W, T, and $h(t)$

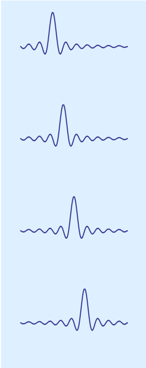
Two orthonormal bases

An orthonormal basis renders the reception finite-dimensional:

Fourier series
(time-limited signal)



Sampling theorem
(bandlimited signal)



Finite-dimensional representation of $h(t)$

Choice of basis:

- Transmitter and receiver must assume the same basis
- We choose the Fourier series

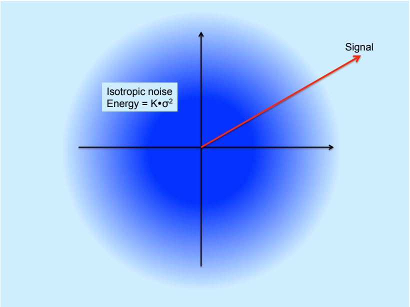
Dimensionality of basis:

- Degrees of freedom (DOF) is $K = W \cdot T$

Regardless of basis:

- Noise is completely random and isotropic

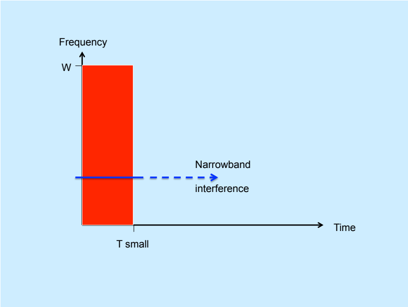
Isotropic noise



- Matched filter looks in the signal direction
- Sensitivity depends on $\mathcal{E}_s$ and σ^2...
- ...and not W, T, and the "shape" of $h(t)$

Radio-frequency interference

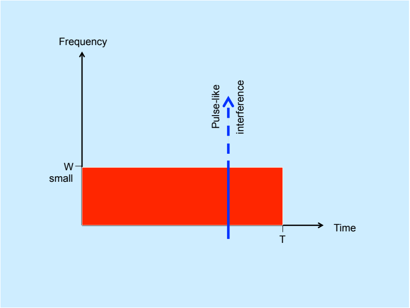
- How to best deal with interference depends on its characteristics
- Narrowband case:



- Want signal energy uniformly distributed over $0 \leq f \leq W$
- Interference overlap $\frac{W_I \cdot T}{W \cdot T} = \frac{W_I}{W}$
- Want W large; T doesn't matter

Interference

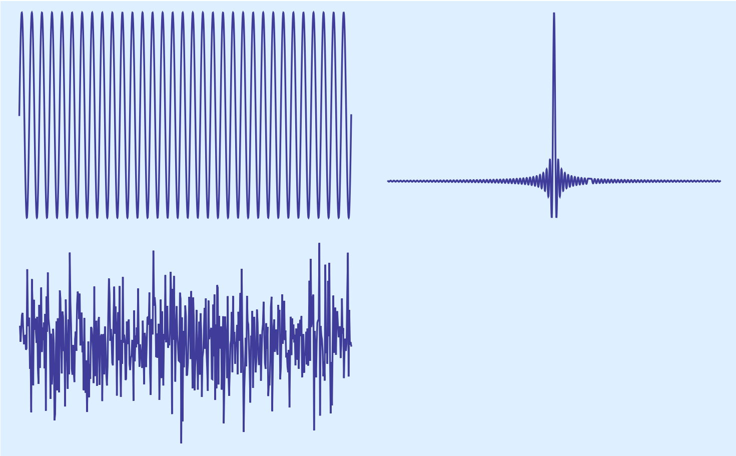
Broadband interference:



- Want signal energy uniformly distributed over $0 \leq t \leq T$
- Interference overlap $\frac{W \cdot T_I}{W \cdot T} = \frac{T_I}{T}$
- Want T large; W doesn't matter

Ways to distribute signal energy

Carrier-like Pulse-like



Noise-like

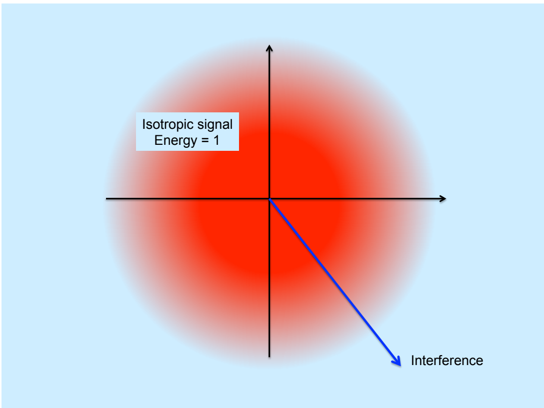
Random signal

If the signal is chosen from a random ensemble, it should be completely random and therefore isotropic

- Statistically, signal component in direction of any interference vector has energy $\mathcal{E}_s/(W \cdot T)$
- Spread spectrum: Make $K = W \cdot T$ large

Current and past SETI Cyclops searches ignore this type of signal

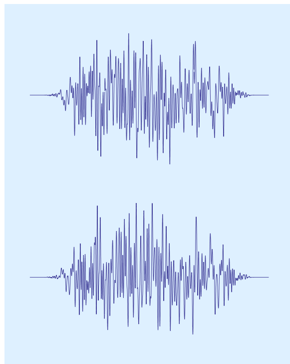
Isotropic signal



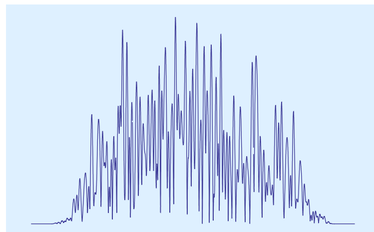
Pseudo-random signal

Binary expansion of π, e, or $\sqrt{2}$

Real and imaginary



Magnitude



Some environmental factors

Time-invariant
- Plasma dispersion
- Scattering

Time-varying
- Doppler
- Turbulence
- Scintillation (fading)

Bandwidth stress test of the ISM

High data rate. $W \cdot T \approx 1$ and $1/T$ large
Spread spectrum. $W \cdot T >> 1$

We choose spread spectrum:

- Suppresses interference
- Usually less affected by multipath
- Discovery is easier
- ISM bandwidth is "free"

Plasma dispersion

- The ISM is conductive due to ionization in interstellar gas clouds
- Homogeneous refractive index

$$n = \left(1 - \left(\frac{f_p}{f}\right)^2\right)^{-1/2}$$

- Frequency-dependent excess group delay

$$\tau(f) = \frac{\mathcal{D} \cdot \mathrm{DM}}{f^2}$$

Relation of group delay and phase

Frequency response:

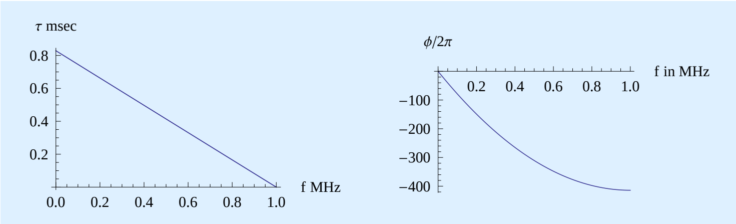
$$F(f) = |F(f)| \cdot e^{i\phi(f)}$$

Monochromatic phase shift:

$$2\pi \cdot \tau(f) = -\frac{\mathrm{d}\phi(f)}{\mathrm{d}f}$$

Typical case

Group delay changes linearly and phase quadratically



$f_c = 1$ GHz, $W = 1$ MHz, DM $= 100$

Delay spread

- Range of group delays across $f_c \le f \le f_c + W$

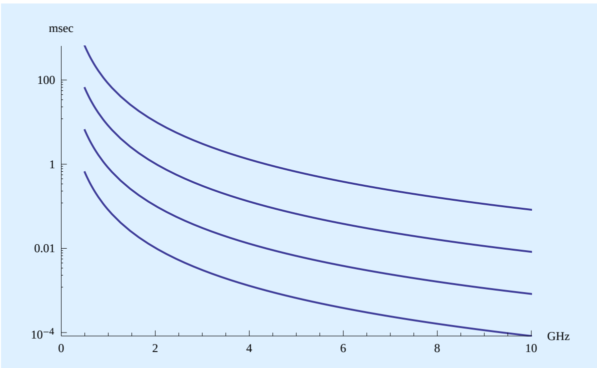
$$\tau_{\max} = \tau(f_c) - \tau(f_c + W)$$

- A priori knowledge from pulsar observations

$$\mathrm{DM}_{\min} \le \mathrm{DM} \le \mathrm{DM}_{\max}$$

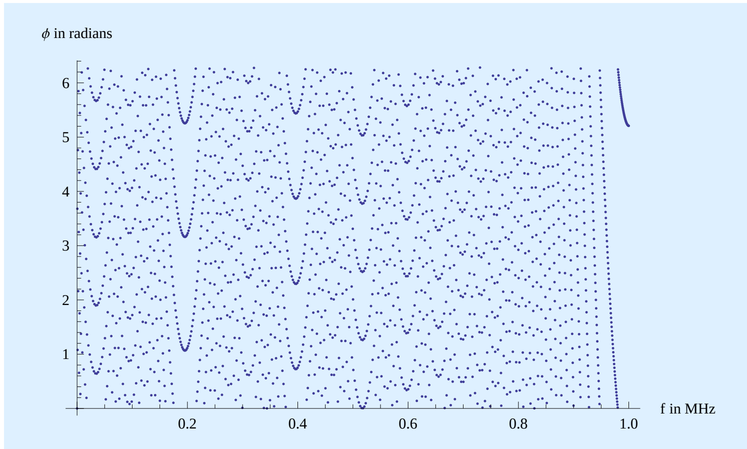
Delay spread vs f_c

Dispersion favors large f_c: $\tau_{\max} \sim f_c^{-3}$



$W = 1$ MHz, DM $= 1,\ 10,\ 100,\ 1000$

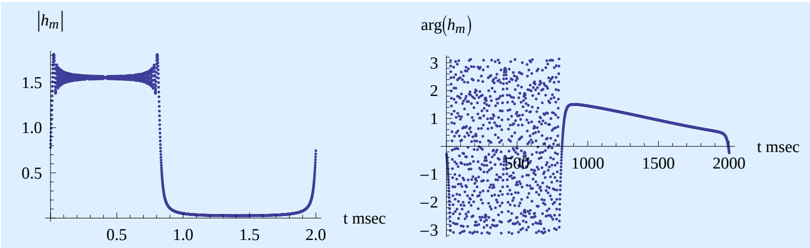
Phase after wrapping



$f = m/T,\ T = 2$ msec$,\ 0 \le m < 2000$

Impulse response

Impulse response energy is spread uniformly over $0 \leq t \leq \tau_{\max}$ but phase is chaotic



$\mathrm{DFT}^{-1}\{e^{i\phi_m}\}$ and $\tau_{\max} \approx 0.8$ msec

Fourier-series representation of $h(t)$

- Fourier-series basis is natural for characterizing dispersion

$$h(t) = \sqrt{\mathcal{E}_s} \cdot w(t) \cdot \frac{1}{\sqrt{K}} \sum_{m=0}^{K-1} c_m \cdot e^{i2\pi m t/T}$$

- Search over T assuming knowledge of $\{c_m\}$
- Less need to search over $K = W \cdot T$ and $w(t)$

Effect of delay spread on one component of $h(t)$

Assuming $\phi(f) \approx$ linear for $f \approx f_0$

$$w(t)\, e^{i 2\pi f_0 t} \longrightarrow \boxed{|F(f)|\, e^{i\phi(f)}} \longrightarrow e^{i\phi(f_0)}\, w\left(t - \tau(f_0)\right)\, e^{i 2\pi f_0 t}$$

- Ignore effect of group delay on $w(t)$ if $\tau_{\max} << T$
- Search over $T >> \tau_{\max}$ as based on $\{f_c,\, W, \mathrm{DM}_{\max}\}$

Performance metric

- What increase in $\mathcal{E}_s$, as a consequence of dispersion, is required to maintain fixed P_{FA} and P_{D}?

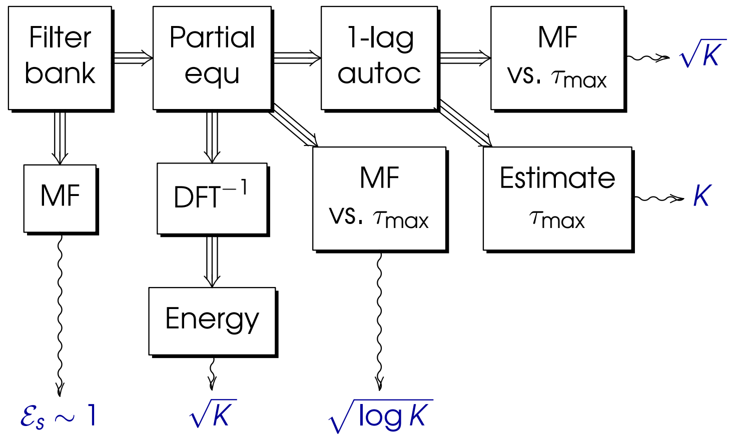
$$\mathcal{E}_s \sim f(K) \quad \text{means} \quad \mathcal{E}_s \approx \alpha \cdot f(K) \;\; \text{for large } K$$

- In terms of power $\mathcal{P}_s$, always favorable to increase T

$$\mathcal{P}_s = \frac{\mathcal{E}_s}{T} \approx \frac{\alpha \cdot f(W \cdot T)}{T} \quad \text{for large } K$$

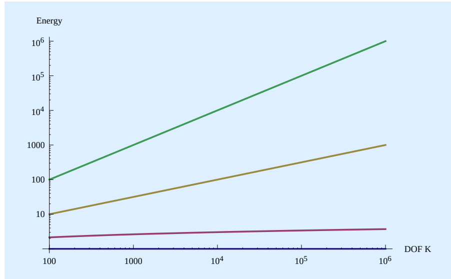
- Always unfavorable to increase W

Processing path options



$\{ \Longrightarrow \} \equiv$ vector transfer

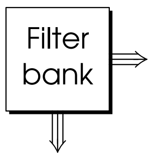
Energy penalty



Increase in $\mathcal{E}_s$ required to maintain P_{FA} and P_D

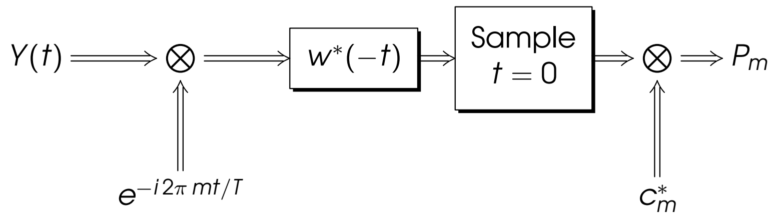
- $\mathcal{E}_s \sim \{1, \sqrt{\log K}, \sqrt{K}, K\}$
- At $K = 10^6$, $\mathcal{E}_s \sim \{1, 3.7, 10^3, 10^6\}$

Filter bank



Filter bank and de-spreading

One channel (out of K):

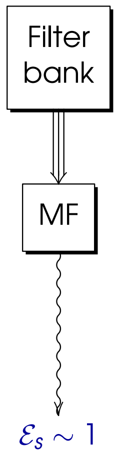


$(\Longrightarrow) \equiv$ complex value transfer

$$P_m = \left(\sqrt{\frac{\mathcal{E}_s}{K}} + O_m \right) \cdot e^{i\phi_m}$$

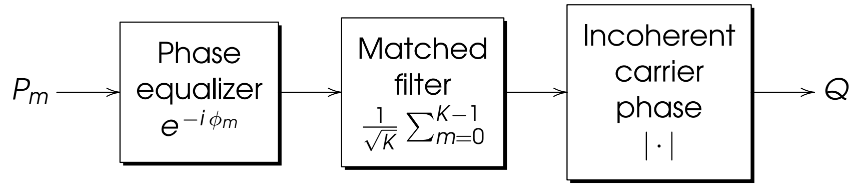
$$E\,|O_m|^2 = \sigma^2$$

Incoherent matched filter



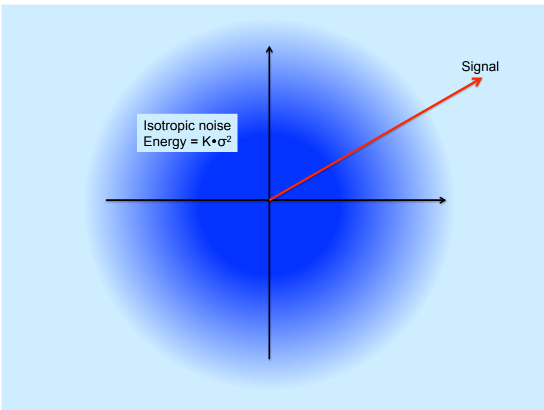
Incoherent matched filter

Assuming $\tau_{\max}$ (hence $\{\phi_m\}$) is supplied by a genie:

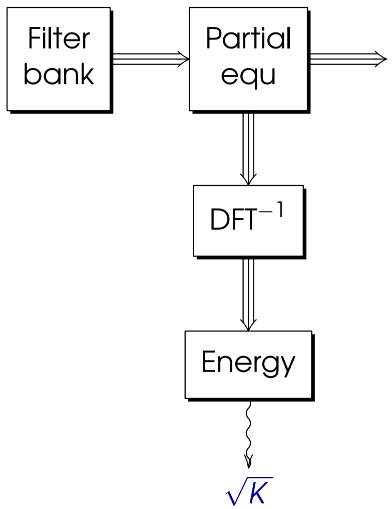


$\mathcal{E}_s \sim 1$

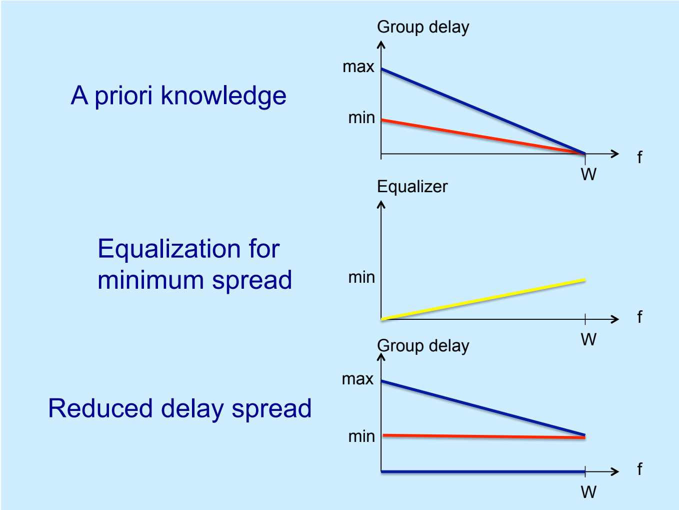
Isotropic noise again



Energy estimation

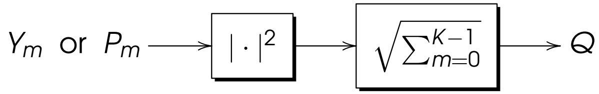


Partial equalization for minimum delay spread



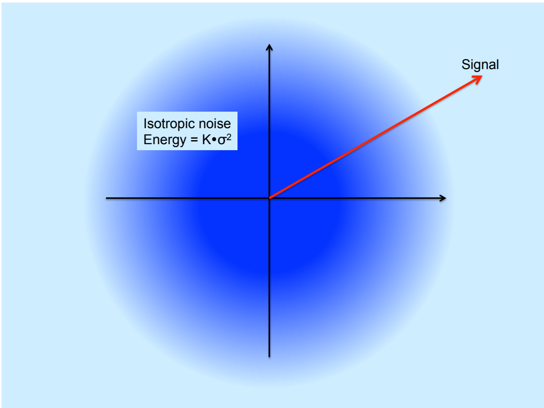
Detection based on energy estimation

Estimating "raw" $\mathcal{E}_s$ does not require knowledge of $\tau_{\max}$ or $\{c_m\}$:

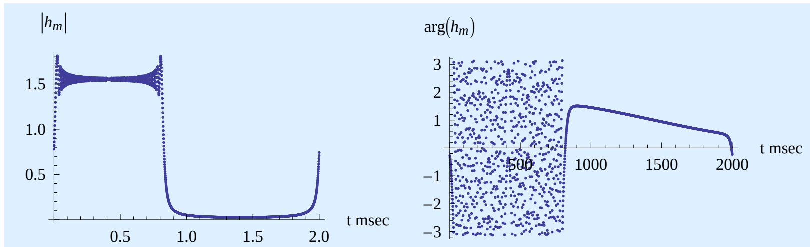


$$\mathcal{E}_s \sim \sqrt{K}$$

Isotropic noise again

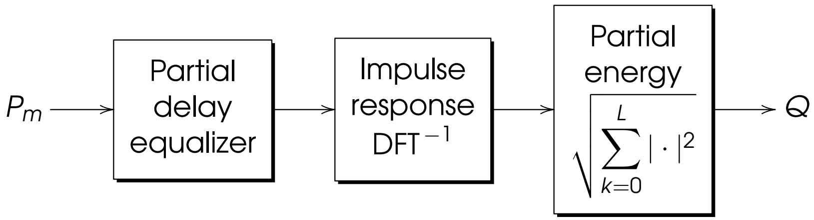


Maximum delay spread



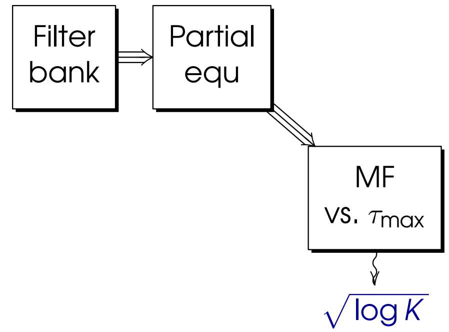
- For specific f_c, W, and LOS $\tau_{\max}$ is bounded

Restricted-delay spread energy estimation



$$\mathcal{E}_s \sim \sqrt{\tau_{\max} \cdot W}$$

Maximum likelihood



Maximum likelihood

- Signal subspace has dimension $L < K$:

$$\mathbf{d}_{\tau_{\max}} = \begin{bmatrix} e^{i\phi_0} \\ e^{i\phi_2} \\ e^{i\phi_{K-1}} \end{bmatrix} \quad \text{for} \quad 0 \leq \tau_{\max} \leq \max_{\mathrm{DM}} \tau_{\max}$$

- Turns out:

$$L \approx \frac{1}{2}\left(\frac{\tau_{\max}}{T}\right) \cdot K$$

- Find orthonormal basis $\{\mathbf{e}_k,\ 1 \leq k \leq L\}$

Maximum likelihood (con't)

If projection of any $\mathbf{d}_{\tau_{\max}}$ is entirely in direction of one basis $\mathbf{e}_k$, then it suffices to perform L independent trials:

- $Q_n = \text{IMF for } \mathbf{e}_m$
- Threshold input $= \max_n Q_n$
- $\mathcal{E}_s \sim \sqrt{\log L}$

Finding orthogonal basis

Singular value decomposition (SVD):

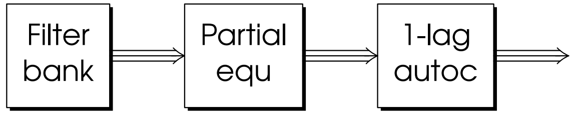
$$\mathbf{D} = \left[\mathbf{d}_1\,\mathbf{d}_2\,\ldots\,\mathbf{d}_K\right] = \mathbf{U}\,\Sigma\,\mathbf{V}^\dagger$$

U is a candidate for orthonormal basis:

$$\mathbf{U}^\dagger\,\mathbf{D} = \Sigma\,\mathbf{V}^\dagger$$

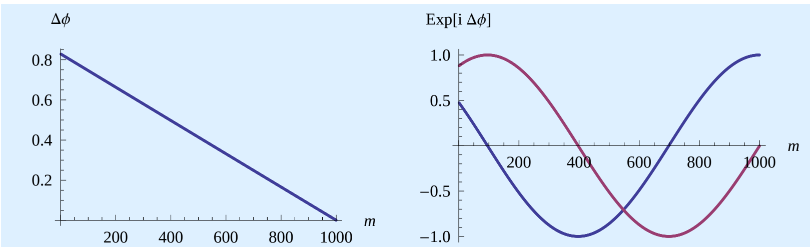
$\tau_{max} \leq T$, $K = 50$, $L = 26$

Autocorrelation



One-lag autocorrection

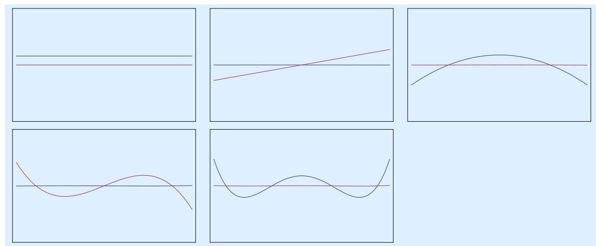
$$\Delta\phi_m = \phi_{m+1} - \phi_m \approx -\frac{2\pi}{T}\cdot\tau\left(\frac{m}{T}\right)$$



$$P_{m+1}P_m^* = \left(\sqrt{\frac{\mathcal{E}_s}{K}} + O_{m+1}\right)\left(\sqrt{\frac{\mathcal{E}_s}{K}} + O_m\right)^* \cdot e^{i\Delta\phi_m}$$

Nonlinear reduction in DOF

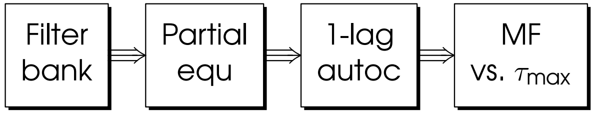
- $\{e^{i\Delta\phi_m},\ 0 \leq m < K\}$ is always less than one period of a complex exponential



$L = 5$ orthonormal basis functions

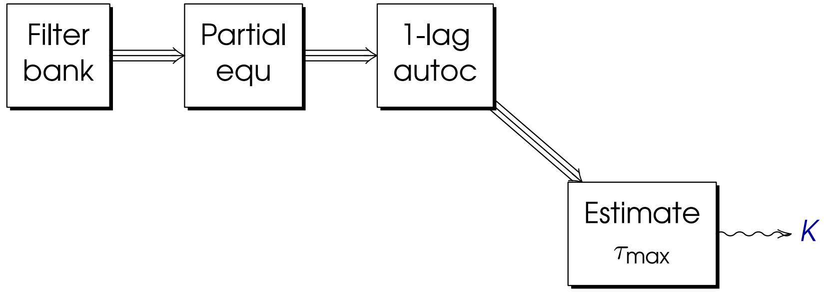
- $L = 2$ usually suffices

Matched filtering after autocorrelation



- $\mathcal{E}_s \sim \sqrt{K}$ (same as energy estimator)
- Results from the autocorrelation noise-on-noise $O_{m+1}O_m^*$ term

Direct estimation of $\tau_{\max}$



Estimation of dispersion

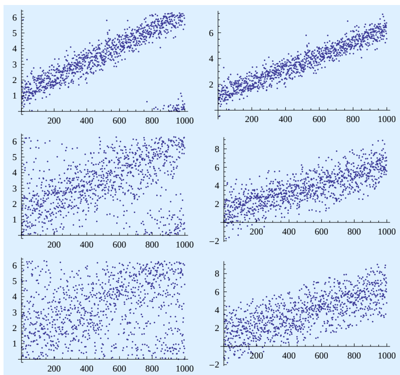
$$\arg\left(P_{m+1}P_m^*\right) = \left(\Delta\phi_{m+1} + \Theta_{m+1} - \Theta_m\right) \mod 2\pi$$

$$\Theta_m = \arg\left(\sqrt{\frac{\mathcal{E}_s}{K}} + O_m\right)$$

- Slope of $\Delta\phi_m$ vs m is proportional to $\tau_{\max}$

Phase estimation and unwrapping

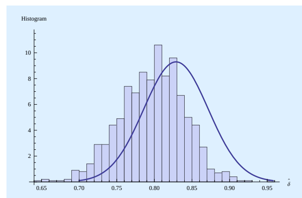
$\arg\left(P_{m+1}P_m^*\right)$



$$\frac{\mathcal{E}_s}{\sigma^2} = \alpha \cdot K$$

$K = 1000$ (30 dB)

$\alpha = 10, 3,$ and 0 dB



Sensitivity of τ_{max} estimation

- $\mathcal{E}_s \sim K$ (much less sensitive then energy estimate)
- Otherwise $\Theta_m \to$ uniform distribution on $[0, 2\pi]$

$$\Theta_m = \arg\left(\sqrt{\frac{\mathcal{E}_s}{K}} + O_m\right) \mod 2\pi$$

Principal tradeoffs

$\uparrow f_c$	Good	$\downarrow \tau_{max} \sim f_c^{-3}$
$\uparrow T$	Good	$\downarrow \tau_{max}/T$ $\downarrow \mathcal{P}_s = \mathcal{E}_s/T$ $\downarrow$ Broadband interference
	Bad	$\downarrow$ Data rate $\sim 1/T$ $\uparrow \mathcal{E}_s \sim \sqrt{\log W \cdot T}$ $\uparrow$ Susceptibility to time-variation
$W \uparrow$	Good	$\downarrow$ Narrowband interference
	Bad	$\uparrow \tau_{max} \sim W$ $\uparrow \mathcal{E}_s \sim \sqrt{\log W \cdot T}$

Takeaways

What to look for:

- The more a priori knowledge of the signal, the more sensitive its detection
  - Conversely, high-sensitivity searches target a specific signal
- Optimization provides implicit design coordination in the form of guidance on the class of signal to use, and suggests spread spectrum

Takeaways (con't)

Where to look:

- Environmental impairments helpfully constrain search parameters
- Detection sensitivity near fundamental limits with reasonable computational burden and high search rate are technologically feasible today for spread spectrum signals with relatively large f_c and large T

Takeaways (con't)

How you can help:

- Communication engineering is immediately relevant to the exciting quest to find life elsewhere in our Universe
- Visit setiquest.org

Postscript

Thanks to:

- SETI Institute: Samantha Blair, Gerry Harp, Jill Tarter, Rick Standahar and Kent Cullers
- National Aeronautics and Space Administration

Further information

My homepage:
`www.eecs.berkeley.edu/~messer`