EECS 225A Spring 2005

Homework 2

Due: February 3. Solutions will be presented on that date and you will self-grade your homework.

Note: In all homework problems you are encouraged to use the numeric and/or symbolic capabilities of Matlab or similar facility.

1. Consider a complex signal \( \{z_k, 1 \leq k \leq n\} \). The goal is to find the best approximation to this signal in terms of a complex exponential with some fixed frequency \( \omega \); that is, the complex coefficients \( u \) and \( v \) such that \( \{u \cdot e^{i\omega k} + v, 1 \leq k \leq n\} \) is the best approximation to \( z_k \) in the sense of minimizing the mean-square error
   \[
   E = \sum_{k=1}^{n} |z_k - u \cdot e^{i\omega k} - v|^2.
   \]
   a. Find a set of sufficient conditions for a stationary point of \( E \).
   b. Find an expression for the coefficients and resulting error suitable for computation.
   c. Compute the resulting coefficients numerically for signal
      \( \{e^{i\omega k + j\pi/4} + x_k + jy_k, 1 \leq k \leq n\} \) where \( \{x_k\} \) and \( \{y_k\} \) are unit-variance real-valued white and independent Gaussian processes generated by a random number generator and \( \omega = \pi/12, n = 1000 \). Interpret the results.
      Repeat the calculation for a larger and smaller variance and interpret how they change.
   d. Repeat c. for signal \( \{e^{i2\omega k + j\pi/4} + x_k + jy_k, 1 \leq k \leq n\} \). Note that there is now a mismatch between the signal frequency and the model frequency.

2. Give an example of a rational transfer functions and associated ROC with each of the following properties. If no such rational transfer function exists, so state and give your reasoning.
   b. Anti-causal real-valued unit-sample response.
   c. Causal imaginary-valued unit-sample response.
   d. Anti-causal imaginary-valued unit-sample response.
   e. Causal unit-sample response that is neither pure real-valued nor pure imaginary valued.
   f. Real-valued unit-sample response that is two-sided.
   g. Real-valued on the unit circle, with both positive and negative values.
   h. Imaginary-valued on the unit circle.
   i. Non-negative real-valued on the unit circle.

3. Give an intuitive argument for the following statement. If you are feeling ambitious, offer a proof. For any rational transfer function that is non-negative real-valued on the unit circle, zeros on the unit circle must have even multiplicity (two, four, six, etc.).