

# Average-Case Complexity — Addendum

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In our review paper on average-case complexity [BT06], several attributions were missing.

The discussion of average-case tractability in Section 2 follows the treatment of Impagliazzo [Imp95]. In particular, Impagliazzo introduced the notion of heuristic scheme and proved that the notion of tractability defined by Levin is equivalent to the existence of errorless heuristic schemes.

The discussion of completeness in Section 3 follows the treatment of Goldreich’s notes [Gol97]. The proof that the Bounded Halting problem is complete for  $\langle \text{NP}, \text{PCOMP} \rangle$  presented in Section 3.2 is due to Gurevich [Gur91].

After we present the  $\langle \text{NP}, \text{PCOMP} \rangle$ -completeness of Bounded Halting, we do not discuss further completeness results, of which a few are known, including the Tiling problem studied in Levin’s [Lev86] foundational paper, a graph coloring problem [VL88, LV18], a matrix decomposition problem [Gur90, BG95], a bounded version of the Post correspondence problem [Gur91], and diophantine matrix problems [VR92].

In the years following the publication of Levin’s [Lev86] paper on completeness in average-case complexity, several researchers contributed to the goal of explaining and generalizing Levin’s ideas, including Johnson [Joh84], Gurevich and McCauley [GM87], and Goldreich [Gol88]. Notions of reducibility between distributional problems, and what notions suffice to prove the Impagliazzo-Levin theorem [IL90] that we present in Section 5, are studied in [BG93].

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