Homework 2

To be finished individually. Due on Tuesday, January 27, 2011. Submit in class, or to GATES460. (sliding under the door if no one is there)

Revised 1/20/2011

1. (6 points) Consider the following linear program, and find all the vertices of its polytope.

\[
\begin{align*}
\text{maximize} & \quad x_1 + x_2 \\
\text{subject to} & \quad 3x_1 + x_2 \leq 3 \\
& \quad x_1 + 3x_2 \leq 3 \\
& \quad x_1 + x_2 \leq 2 \\
& \quad x_1, x_2 \geq 0 
\end{align*}
\]

2. (7 points) Find the dual of the following linear program.

\[
\begin{align*}
\text{maximize} & \quad 6x_1 + 8x_2 + 5x_3 + 9x_4 \\
\text{subject to} & \quad 2x_1 + x_2 + x_3 + 3x_4 > 5 \\
& \quad x_1 + 3x_2 + x_3 + 2x_4 = 3 \\
& \quad x_1, x_2 \geq 0 \\
& \quad x_3, x_4 \text{ are unconstrained}
\end{align*}
\]

3. (7 points) In a facility location problem, there is a set of facilities and a set of cities, and our job is to choose a subset of facilities to open, and to connect every city to some one of the open facilities. There is a nonnegative cost $f_j$ for opening facility $j$, and a nonnegative connection cost $c_{i,j}$ for connecting city $i$ to facility $j$. Given these as input, we look for a solution that minimizes the total cost. Formulate this facility location problem as an integer programming problem, and find its linear programming relaxation.