Problem 1

Def: $\text{OR}_n : \{0,1\}^n \rightarrow \{0,1\}$ is the $n$-bit or function.
Let $p$ be any polynomial such that
\[ \forall x \in \{0,1\}^n : p(x_1, ..., x_n) = \text{OR}_n(x_1, ..., x_n). \]
Prove: $\deg(p) \geq n$

Problem 2

Def: Majority : $\{0,1\}^* \rightarrow \{0,1\}$
\[ \text{ Majority}(x) = \begin{cases} 1 & \text{ if } \sum_{i=1}^n x_i \geq n/2, \text{ where } n = |x| \\ 0 & \text{ otherwise} \end{cases} \]
Prove: Majority can be computed by a family of log depth, poly size, bounded fan-in circuits.