Problem 1 Solution
\[ \left( (01) \cup \left( (1 \cup 00)(10)^* (0 \cup 11) \right) \right)^* (1 \cup 00)(10)^* \]

Problem 2 Solution
Let \( a = (00 \cup 01 \cup 10). \)
The answer is \( \left( (11)^* a (11) \right) \cup a \right)^* (11)a^* \]

Problem 3 Solution
\( (0 \cup 1)^+ 1(0 \cup 1)^3 \)

Problem 4 Solution
A regular expression can be defined as follows (where \( M \) represents \( \Sigma \))

\[
\text{Regex} = \text{Symbol} \ M \\
| \text{Concat} \ \text{Regex} \ \text{Regex} \\
| \text{Choice} \ \text{Regex} \ \text{Regex} \\
| \text{Star} \ \text{Regex} \\
| \text{Null}
\]

this states that a regular expression is one of:

- a symbol of \( \Sigma \)
- concatenation of two regular expressions
- choice of two regular expressions
- or star of a regular expression

Now, consider the following function:

\[
\text{reverse} :: \text{Regex} \rightarrow \text{Regex} \\
\text{reverse} (\text{Symbol} \ x) = \text{Symbol} \ x \\
\text{reverse} (\text{Concat} \ \text{regA} \ \text{regB}) = \text{Concat} (\text{reverse} \ \text{regB}) (\text{reverse} \ \text{regA}) \\
\text{reverse} (\text{Choice} \ \text{regA} \ \text{regB}) = \text{Choice} (\text{reverse} \ \text{regA}) (\text{reverse} \ \text{regB}) \\
\text{reverse} (\text{Star} \ \text{regA}) = \text{Star} (\text{reverse} \ \text{regA}) \\
\text{reverse} (\text{Null}) = \text{Null}
\]

We claim that for all regular expressions \( r \), \( \text{reverse}(r) \) accepts the reverse of the language accepted by \( r \).
The proof of this follows by structural induction on regular expressions.