Problem 1

Prove there exists an infinite sequence of languages $L_1, L_2, \ldots$ such that:

- For every $i$, $L_i$ is neither recognizable nor co-recognizable
- $L = \bigcap_i L_i$ is regular
- $|L| = \infty$
Problem 2
Prove: $NEXP \neq EXP \Rightarrow NP \neq P$
Problem 3

Prove: $P = NP$ iff there exists $k, l > 2$ such that $\text{NTime}(O(n^k)) \subseteq \text{DTime}(O(n^l))$
Problem 4

For $f : \{0, 1\}^n \rightarrow \{0, 1\}$, let $\text{size}(f)$ be the size of the smallest circuit which implements function $f$.

Prove: forall $T(n) \leq 2^n/1000n$, there exists $f : \{0, 1\}^n \rightarrow \{0, 1\}$ such that $T(n) \leq \text{size}(f) \leq T(n) + 10n$.

You can use the fact that for every $n$, there are functions $f : \{0, 1\}^n \rightarrow \{0, 1\}$ that cannot be computed by circuits of size $< 2^n/1000n$. 
Problem 5

Let $K(x)$ be the Kolmogorov complexity of $x$.
Let $X_n$ be the set of all TMs $M$ where (1) $|Q| + |\Gamma| \leq n$ and (2) $M$ halts on the empty input.
Let $BB(n)$ be the longest time any TM in $X_n$ runs on the empty input.

Design an algorithm $A$ such that $A^{BB}(x) = K(x)$. ($A$ is granted a blackbox routine that computes $BB$, and is allowed to query it.)
Problem 6

An Eulerian cycle of an undirected graph is a cycle that visits every edge exactly once.

For every integer $n$, consider the streaming complexity of the problem of deciding whether a graph on $n$ vertices, given by a stream of edges, has an Eulerian cycle.

That is, for a set $n$ vertices $V$, our alphabet $\Sigma = \{\{v_1, v_2\} \mid v_1, v_2 \in V, v_1 \neq v_2\}$ is all possible (undirected) edges between these vertices and our stream is a sequence of these edges. If we call the set of each edge in this stream $E$, then $G = (V, E)$ is the undirected graph defined by it. We want a streaming algorithm that takes the stream and computes whether or not $G$ has an Eulerian cycle. Show that the bits of memory required for a streaming algorithm for this problem is $\Omega(n)$. 
Problem 7

Undirected graphs $G$ and $H$ are isomorphic iff there exist a bijection $f : G_V \rightarrow H_V$ such that $(u, v) \in G_E$ iff $(f(u), f(v)) \in H_E$.

Let $L = \{\langle G, H \rangle | (G, H) \text{ are isomorphic } \}$.

Define a zero-knowledge protocol for $L$.

Prove that it is complete, sound, and perfect zero-knowledge.

*Hint: Prover sends a permuted graph. Verifier sends a bit. Prover sends a permutation.*