Practice Midterm 1

1. State whether each of the following statements is true. In addition, give a short proof (2-3 lines are sufficient) if the statement is true, and give a counterexample otherwise.

   (a) If \( L_1, L_2, \ldots, L_{172} \) are all regular languages, then the language \( \bigcap_{i=1}^{172} L_i \) is regular.

   (b) If \( L_1, L_2, L_3, \ldots \) is an infinite sequence of regular languages, then the language \( \bigcap_{i=1}^{\infty} L_i \) is regular.

2. Let
   \[
   L = \{((D), w) \mid D \text{ is a DFA over the binary alphabet } \{0, 1\} \text{ that accepts } w\}
   \]
   (Assume that the encoding of DFAs also uses the binary alphabet.)

   (a) Show that \( L \) is not regular.

   (b) Show that \( L \) is decidable.

3. Consider the language
   \[
   INT_\text{TM} = \{\langle M_1, M_2 \rangle : L(M_1) \cap L(M_2) \neq \emptyset\}.
   \]
   (Thus, \( INT_\text{TM} \) is the language associated with the problem of deciding whether, for two given Turing machines \( M_1 \) and \( M_2 \), there is some string that is accepted by both machines.)

   (a) Show that \( INT_\text{TM} \) is Turing recognizable.

   (b) Show that \( INT_\text{TM} \) is not decidable.

4. Let \( S = \{\langle M \rangle \mid M \text{ is a TM and } L(M) = \{\langle M \rangle\}\}. \) Show that neither \( S \) nor \( \overline{S} \) is Turing-recognizable.