Problem Set 1

This problem set is due on Wednesday January 31, by 4:00pm.

Use the CS172 drop box.

Write your name and your student ID number on your solution. Write legibly. The description of your proofs should be as clear as possible (which does not mean long – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage www.cs.berkeley.edu/~luca/cs172.

1. Prove that the following languages are regular, either by exhibiting a regular expression representing the language, or a DFA/NFA that recognizes the language:

   (a) all strings that do not contain the substring $aba$, for $\Sigma = \{a, b\}$ (for instance, $aabaa$ contains the substring $aba$, whereas $abba$ does not)

   (b) set of strings such that each block of 4 consecutive symbols contains at least two $a$'s, for $\Sigma = \{a, b\}$

   (c) set of binary strings ($\Sigma = \{0, 1\}$) which when interpreted as a number (with the most significant bit on the left), are divisible by 5.

2. (Sipser, problem 1.31) For any string $w = w_1w_2\cdots w_n$, the reverse of $w$, written as $w^R$ is the string $w$ in reverse order, $w_n\cdots w_2w_1$. For any language $A$, let $A^R = \{w^R \mid w \in A\}$. Show that if $A$ is regular, so is $A^R$.

3. We say a string $w = w_1w_2\cdots w_n$ is a shuffle of strings $u$ and $v$ if there exists $J \subseteq \{1, \ldots, n\}$ such that $(w_j)_{j \in J} = u$ and $(w_j)_{j \notin J} = v$. For example CSS17PR2ING07 is a shuffle of the strings CS172 and SPRING07 and in fact, there are two sets $J = \{1, 2, 4, 5, 8\}$ and $J = \{1, 3, 4, 5, 8\}$ which work here.

We then define the shuffle of two languages $A$ and $B$ as

$$S(A, B) = \{w \exists u \in A, v \in B \text{ s.t. } w \text{ is a shuffle of } u \text{ and } v\}$$

Show that if $A$ and $B$ are regular languages over a common alphabet $\Sigma$, then so is $S(A, B)$. 