1. **The hardest of them all**
   Show that a language $A$ is:
   
   (a) Turing-recognizable iff $A \leq_m A_{TM}$.
   (b) decidable iff $A \leq_m 0^*1^*$.

2. **More on Rice’s theorem**
   In Rice’s theorem, we prove that a language $L$ consisting of Turing machine descriptions such that the language of the TMs belong to a class $C$, is undecidable. We assumed two properties of $L$:
   
   (a) $L$ is nontrivial i.e. $L$ is not empty or equal to the set of all Turing machines.
   (b) If $L(M_1) = L(M_2)$, then $\langle M_1 \rangle \in L \Leftrightarrow \langle M_2 \rangle \in L$.
   
   Prove that both these properties are necessary for proving $L$ to be undecidable.

3. **When recognizability met decidability**
   Let $C$ be a language. Prove that $C$ is Turing-recognizable iff a decidable language $D$ exists such that $C = \{x \mid \exists y (\langle x, y \rangle \in D)\}$.
   
   *Hint*: Think of $y$ as a proof that $x \in C$. What can be a good proof?

4. **Cantor’s ghost**
   Let $S$ be a set and let $P(S)$ be the set of all the subsets of $S$. Show that $|P(S)| > |S|$.
   
   *Hint*: First show this for the set of natural numbers.