1. **The friends...**
Show that the following languages are regular by giving regular expressions for them:

(a) The set of binary strings with at most one pair of consecutive 1s.
(b) The set of binary strings with an equal number of zeroes and ones, such that the difference between the number of zeroes and number of ones never exceeds 2 in any prefix.
(c) The set of all binary strings not containing 101 as a substring.

2. **The foes...**
Prove that the following languages are not regular using the pumping lemma:

(a) \{a^{2^n} \mid n \geq 0\}.
(b) The set of all binary strings which are palindromes (i.e. \(w \in \{0, 1\}^* \mid w = w^R\)).
(c) \(\{0^i1^j \mid \gcd(i, j) = 1\}\).

3. **...and the impostor!**
Consider the language \(F = \{a^ib^jc^k \mid i, j, k \geq 0 \text{ and if } i = 1 \text{ then } j = k\}\).

(a) Show that \(F\) acts like a regular language in the pumping lemma i.e. give a pumping length \(p\) and show that \(F\) satisfies the conditions of the lemma for this \(p\).
(b) Show that \(F\) is not regular.
(c) Why is this not a contradiction?