Problem Set 6

This problem set is due on Friday March 12, by 4:00pm.

Use the CS172 drop box.

Write your name and your student ID number on your solution. Write legibly. The description of your proofs should be as clear as possible (which does not mean long – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage www.cs.berkeley.edu/~luca/cs172.

1. Sipser problem 5.9.

2. Sipser problem 5.12. Prove the result using Rice’s theorem: show that $S = L_C$, for a properly defined $C$, and show that $S$ is non-empty and does not contain all Turing machines.

3. (a) Suppose $B$ is an undecidable language such that $B \leq_m \overline{B}$. Prove that neither $B$ nor $\overline{B}$ is Turing-recognizable.
   
   (b) (Sipser problem 5.11) Give an example of an undecidable language $B$ where $B \leq_m \overline{B}$. (A correct example of such a language without proof will get zero credit; a complete solution should include an explicit mapping reduction from $B$ to $\overline{B}$ and a proof that $B$ is undecidable.)