Problem Set 2

This problem set is due on **Friday February 6, by 4:00pm.**

Use the CS172 drop box.

Write your name and your student ID number on your solution. Write legibly. The description of your proofs should be as clear as possible (which does not mean long – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage [www.cs.berkeley.edu/~luca/cs172](http://www.cs.berkeley.edu/~luca/cs172).

1. Sipser problem 1.41.

2. A string is a palindrome if it reads the same way forward and backward, like radar, or 1101011. Show that for every alphabet \( \Sigma \) (with \( |\Sigma| \geq 2 \)), the language of palindromes over \( \Sigma \) is not regular.

3. (a) Let \( A \) be the set of strings over \( \{0, 1\} \) that can be written in the form \( 1^k y \) where \( y \) contains at least \( k \) 1s, for some \( k \geq 1 \). Show that \( A \) is a regular language.

   [Note that the same string could fit the definition for more than one value of \( k \). For example 1101010 can be seen as 1 followed by the string \( y = 101010 \), which contains at least one 1, or as 11 followed by 01010. On the other hand, the string 100, for example, is not in \( A \) because there is no value of \( k \) for which the definition applies.]

   (b) Let \( B \) be the set of strings over \( \{0, 1\} \) that can be written in the form \( 1^k 0 y \) where \( y \) contains at least \( k \) 1s, for some \( k \geq 1 \). Show that \( B \) is not a regular language.

   (c) Let \( C \) be the set of strings over \( \{0, 1\} \) that can be written in the form \( 1^k z \) where \( z \) contains at most \( k \) 1s, for some \( k \geq 1 \). Show that \( C \) is not a regular language.

4. Let \( k \) be a positive integer. Let \( \Sigma = \{0, 1\} \), and \( L \) be the language consisting of all strings over \( \{0, 1\} \) containing a 1 in the \( k \)th position from the end (in particular, all strings of length less than \( k \) are not in \( L \)).

   (a) Prove that any DFA that recognizes \( L \) has at least \( 2^k \) states.

   (b) Prove that any NFA that recognises \( L \) has at least \( k \) states.