Problem Set 12

This problem set is due on Tuesday May 11, by 4:00pm.
Use the CS172 drop box.

Write your name and your student ID number on your solution. Write legibly. The description of your proofs should be as clear as possible (which does not mean long – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage www.cs.berkeley.edu/~luca/cs172.

1. Prove a hierarchy theorem for circuit size. For concreteness, show that there is a constant $c$ such that for every sufficiently large $n$ there is a Boolean function $f : \{0,1\}^n \rightarrow \{0,1\}$ such that $f \not\in \text{SIZE}(n^3)$, but $f \in \text{SIZE}(cn^3 \log n)$.
   [Note: you might be able to show $f \in \text{SIZE}(cn^3)$, or even $f \in \text{SIZE}(n^3 + cn)$. There is a construction that achieves $f \in \text{SIZE}(n^3 + c)$.

2. Show that there is a language $L$ that can be solved by a family of read-once branching programs of size $O(n)$ but such that any automaton that decides $L \cap \{0,1\}^n$ must have $2^{\Omega(n)}$ states.

3. Sipser Problem 10.11.