Problem Set 11

This problem set is due on **Friday Apr 30, by 4:00pm.**

Use the CS172 drop box.

Write your name and your student ID number on your solution. Write legibly. The description of your proofs should be as clear as possible (which does not mean long – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage [www.cs.berkeley.edu/~luca/cs172](http://www.cs.berkeley.edu/~luca/cs172).

(This version was posted on 4/26/04.)

1. Define

   \[ A_{\text{LBA}} = \{ (M, w) \mid M \text{ is a linear bounded automaton (LBA)} \]

   \[ \text{with tape alphabet of size four that accepts } w \} \]

   (Refer to Sipser page 177 for the definition of a LBA; it is essentially a Turing machine that uses linear space.)

   (a) Prove that \( A_{\text{LBA}} \) is \( \text{PSPACE} \)-complete under logarithmic space reductions, and deduce that \( A_{\text{LBA}} \) is also \( \text{PSPACE} \)-complete under polynomial-time reductions.

   (b) Prove that \( A_{\text{LBA}} \notin \text{NL} \).

   (c) Prove that \( A_{\text{LBA}} \in \text{SPACE}(O(n)) \).

   (d) Prove that \( \text{P} \neq \text{SPACE}(O(n)) \).

   [Note: do not try to prove \( A_{\text{LBA}} \notin \text{P} \), which is probably true but hopeless. Instead, assume \( \text{P} = \text{SPACE}(O(n)) \) and then, using the previous results and other things you know about \( \text{P} \), deduce a contradiction.]

2. Sipser 9.18. Correct the last two lines as in the Errata, so that it reads

   \[ \text{pad}(A, f(n)) = \{ \text{pad}(s, f(n)) \mid s \in A, \text{ where } n \text{ is the length of } s \} . \]

   Prove that if \( A \in \text{TIME}(O(n^6)) \) then \( \text{pad}(A, n^2) \in \text{TIME}(O(n^3)) \).

3. Sipser 9.19. Define \( \text{EXPTIME} = \text{TIME}(2^{O(n)}) \) and \( \text{NEXPTIME} = \text{NTIME}(2^{n^{O(1)}}) \).

   Show that \( \text{EXPTIME} \neq \text{NEXPTIME} \) implies \( \text{P} \neq \text{NP} \).