Problem Set 1

This problem set is due on Friday January 30, by 4:00pm.

Use the CS172 drop box.

Write your name and your student ID number on your solution. Write legibly. The description of your proofs should be as clear as possible (which does not mean long – in fact, typically, good clear explanations are also short.) Be sure to be familiar with the collaboration policy, and read the overview in the class homepage www.cs.berkeley.edu/~luca/cs172.

1. Prove that the following languages are regular, either by exhibiting a regular expression representing the language, or a DFA/NFA that recognizes the language:

   (a) the set of all words in the Oxford English dictionary, for \( \Sigma = \{a,b,\ldots,z\} \)

   (b) all strings that do not contain the substring \( aba \), for \( \Sigma = \{a,b\} \) (for instance, \( aabaa \) contains the substring \( aba \), whereas \( abba \) does not)

   (c) all strings that do not contain 3 consecutive occurrences of the same letter, for \( \Sigma = \{a,b\} \)

2. (Sipser, problem 1.24) For any string \( w = w_1w_2\cdots w_n \), the reverse of \( w \), written as \( w^R \) is the string \( w \) in reverse order, \( w_n\cdots w_2 w_1 \). For any language \( A \), let \( A^R = \{ w^R \mid w \in A \} \). Show that if \( A \) is regular, so is \( A^R \).

3. For any language \( A \) with alphabet \( \Sigma \), let

   \[ A_{\text{sub}} = \{ w \in \Sigma^* \mid w \text{ is a substring of } x, \text{ for some } x \in A \} \]

   Show that if \( A \) is regular, so is \( A_{\text{sub}} \).

4. Let \( k \) be a positive integer. Let \( \Sigma = \{0,1\} \), and \( L \) be the language consisting of all strings over \( \{0,1\} \) containing a 1 in the \( k \)th position from the end (in particular, all strings of length less than \( k \) are not in \( L \)).

   (a) Construct a DFA with exactly \( 2^k \) states that recognizes \( L \).

   (b) Construct a NFA with exactly \( k + 1 \) states that recognizes \( L \).