Practice Midterm 2

1. Consider the language

$$INT_{TM} = \{ \langle M_1, M_2 \rangle : L(M_1) \cap L(M_2) \neq \emptyset \}.$$

(Thus, $INT_{TM}$ is the language associated with the problem of deciding whether, for two given Turing machines $M_1$ and $M_2$, there is some string that is accepted by both machines.)

(a) Show that $INT_{TM}$ is Turing recognizable.

(b) Show that $INT_{TM}$ is not decidable.

(c) Is the language

$$EINT_{TM} = \{ \langle M_1, M_2 \rangle : L(M_1) \cap L(M_2) = \emptyset \}.$$

Turing recognizable?

2. Consider the following time-bounded variant of Kolmogorov complexity, written $K_L(x)$, and defined to be the shortest string $\langle M, w, t \rangle$ where $t$ is a positive integer written in binary, and $M$ is a TM that on input $w$ halts with $x$ on its tape without $t$ steps.

(a) Show that $K_L(x)$ is computable (by describing an algorithm that on input $x$ outputs $K_L(x)$).

(b) Prove that for all positive integers $n$, there exists a string $x$ of length $n$ such that $K(x) = O(\log n)$ and $K_L(x) \geq n$. (In fact, there is an algorithm that on input $n$ finds such a $x$.)

3. (Sipser 7.36) For a cnf-formula $\phi$ with $m$ variables and $c$ clauses (that is, $\phi$ is the AND of $c$ clauses, each of which is an OR of several variables), show that you can construct in polynomial time an NFA with $O(cm)$ states that accepts all nonsatisfying assignments, represented as Boolean strings of length $m$. Conclude that the problem of minimizing NFAs (that is, on input a NFA, find the NFA with the smallest number of states that recognizes the same language) cannot be done in polynomial time unless $P = NP$.

4. Prove that the halting problem $HALT_{TM}$ for Turing machines is $NP$-hard.