Problem 1

For this question, $\Sigma = \{a, b\}$.

a. Let Let $L = \{w | w \in \Sigma^*, w \text{ does not end in } aba\}$.

- Write a regular expression for L. (Submit this online : *HW6_1a*)
- Give a short (1-2 sentences) justification for the logic behind the regular expression.

b. Let $L = \{w | w \in \Sigma^*, \text{the third symbol of } w \text{ is } a\}$.

- Write a regular expression for L. (Submit this online : *HW6_1b*)
- Give a short (1-2 sentences) justification for the logic behind the regular expression.

Solution (16 points)

a. Let Let $L = \{w | w \in \Sigma^*, w \text{ does not end in } aba\}$.

- `((a|b)*(aaa|aab|abb|baa|bab|bba|bbb))|(a|b|ε)(a|b|ε)`
- The first part of this regular expression generates all strings w with $|w| \geq 3$ that don't end in *aba*. The second part of the regular expression generates all strings w with $|w| < 3$, which by definition don't end in *aba*.

Test strings : Positives : *abbababababba*, *ababbababb*, ε, *a*, *aa*, *ab* ; Negatives : *abbabbaba*, *aba*

b. Let $L = \{w | w \in \Sigma^*, \text{the third symbol of } w \text{ is } a\}$.

- `(a|b)(a|b)a(a|b)*`
- This regular expression generates strings with either a or b in the first and second positions, a in the third position, and any number of characters after this a.

Test strings : Positives : *abababab*, *aaa*, *aba* ; Negatives : ε, *a*, *ab*, *aaba*, *aa*, *bb*

Problem 2

For this question, $\Sigma = \{a, b\}$.

a. Let $L = \{w | w \in \Sigma^*, w \text{ does not contain } bb \text{ as a substring}\}$.

- What is the minimum number of states that a DFA to recognise L must have? Give a representative string from each equivalence class.
- Write a regular expression for L. (Submit this online : *HW6_2a*)
- Give a short (1-2 sentences) justification for the logic behind the regular expression.

b. Let $L = \{w | w \in \Sigma^*, w \text{ has an odd number of } a\text{s and starts and ends with a } b\}$.

- What is the minimum number of states that a DFA to recognise L must have? Give a representative string from each equivalence class.
- Write a regular expression for L. (Submit this online : *HW6_2b*)
- Give a short (1-2 sentences) justification for the logic behind the regular expression.

Solution (30 points)

For this question, $\Sigma = \{a, b\}$.

a. Let $L = \{w | w \in \Sigma^*, w \text{ does not contain } bb \text{ as a substring}\}$.

- A DFA to recognise L must have at least 3 states. The equivalence classes are $[\varepsilon]$ (accepting), $[b]$ (accepting), $[bb]$ (rejecting).
- `(a|ba)*(`ϵ`|b)`
- The first part of this regular expression generates all strings in the equivalence class of $[\varepsilon]$ with respect to L. The second part can append a b to any such string, generating a string in the equivalence class of $[b]$ with respect to L.

Test strings : Positives : $aaaa$, ε, $baaaaaaba$; Negatives : $babba$, bb

b. Let $L = \{w | w \in \Sigma^*, w \text{ has an odd number of } a\text{s and starts and ends with a } b\}$.

- A DFA to recognise L must have at least 5 states. The equivalence classes are $[\varepsilon]$ (rejecting), $[a]$ (rejecting), $[b]$ (rejecting), $[ba]$ (rejecting), $[bab]$ (accepting).
- `b(b|ab*a)*abb*`
- The bs on either end of the regular expression ensure that the string starts and ends with a b. There is one compulsory a in every string, and additional as are introducted in pairs with unlimited intervening bs.

Test strings : Positives : $babaab$, $babbbb$, $bbbaabbbaaabbb$; Negatives : ε, bb

Problem 3

For this question, $\Sigma = \{a, b\}$.

a. Let $L = \left\{a^n b^{n^2} | n \in \mathbb{N}\right\}$. Use the Myhill Nerode theorem to prove that L is not regular.

b. Let $L = \{w | w \in \Sigma^*, w = w^R\}$ [1]. Use the Myhill Nerode theorem to prove that L is not regular.

Solution (20 points)

For this question, $\Sigma = \{a, b\}$.

a. Using the Myhill-Nerode theorem, we prove that the language $L = \left\{a^n b^{n^2} | n \in \mathbb{N}\right\}$ is not regular.

Consider the set of strings $S = \{a^i | i \in \mathbb{N}, i \geq 0\}$. This is an infinite set of strings. Let $w_i = a^i$ and $w_j = a^j$ be two arbitrary strings in S such that $i \neq j$. Append the string $x = b^{i^2}$ to each of w_i and w_j. Since $i \neq j$, $w_i x = a^i b^{i^2} \in L$, but $w_j x = a^j b^{i^2} \notin L$.

Since the strings were chosen arbitrarily, any two strings in the infinite set S are distinguishable with respect to L. By the Myhill-Nerode theorem, L is not regular.

b. Using the Myhill-Nerode theorem, we prove that the language $L = \{w | w \in \Sigma^*, w = w^R\}$ is not regular.

Consider the set of strings $S = \{a^i b | i \in \mathbb{N}, i \geq 0\}$. This is an infinite set of strings. Let $w_i = a^i b$ and $w_j = a^j b$ be two arbitrary strings in S such that $i \neq j$. Append the string $x = a^i$ to each of w_i and w_j. $w_i x = a^i b a^i = (w_i x)^R$, so $w_i x \in L$, but $w_j x = a^j b a^i \neq a^i b a^j = (w_i x)^R$, so $w_j x \notin L$.

Since the strings were chosen arbitrarily, any two strings in the infinite set S are distinguishable with respect to L. By the Myhill-Nerode theorem, L is not regular.

[1] w^R is w in reverse.

Problem 4

Let $L = \{w \in \{0, 1, 2\}^* | w$ contains the same number of copies of the substrings *01* and *10*$\}$. Is L regular? If so, give a regular expression for L (Submit this online : *HW6_4opt* - optional, of course). If not, use the Myhill Nerode theorem to prove that L is not regular.

Solution (14 points)

Using the Myhill-Nerode theorem, we prove that the language
$L = \{w \in \{0, 1, 2\}^* | w$ contains the same number of copies of the substrings *01* and *10*$\}$ is not regular.

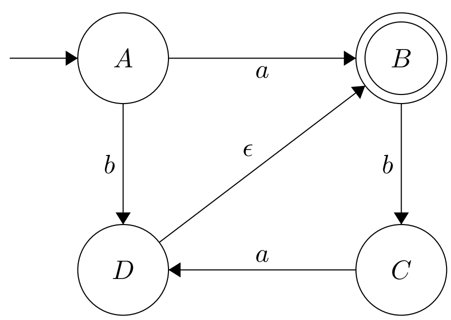
Consider the set of strings $S = \{(012)^i | i \in \mathbb{N}, i \geq 0\}$. This is an infinite set of strings. Let $w_i = (012)^i$ and $w_j = (012)^j$ be two arbitrary strings in S such that $i \neq j$. The string *01* appears i and j times respectively in w_i and w_j.

Append the string $x = (102)^i$, which contains i copies of the string *10*, to each of w_i and w_j. $w_i x = (012)^i (102)^i \in L$, but $w_j x = (012)^j (102)^i \notin L$.

Since the strings were chosen arbitrarily, any two strings in the infinite set S are distinguishable with respect to L. By the Myhill-Nerode theorem, L is not regular.

Problem 5

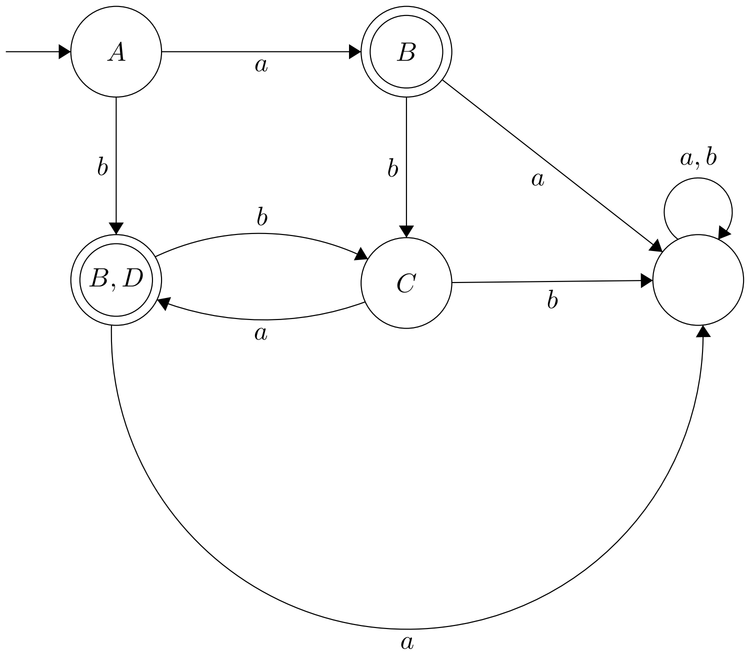
a. – Convert the following NFA to a DFA using the subset construction. (Submit the resulting DFA online : *HW6_5a*)
   – List the subsets of $\{A, B, C, D\}$ that correspond to states in the constructed DFA.



b. Minimise the resulting DFA. (Submit the minimised DFA online : *HW6_5b*)

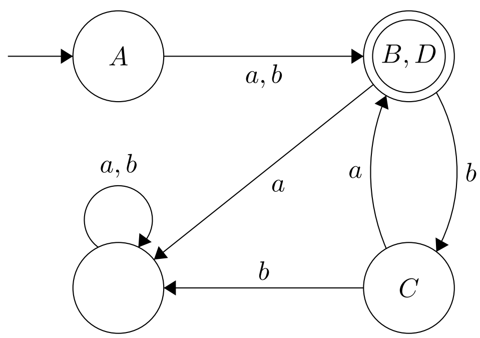
Solution (20 points)

a. The subsets are $\{A\}$, $\{B\}$, $\{C\}$, $\{B, D\}$, and $\{\}$.



Test strings : Accept a, b, aba, bba ; Reject ab, abb, ε

b.



Test strings : Accept a, b, aba, bba ; Reject ab, abb, ε