NONLINEAR STANDING WAVE EXCITATION
BY SERIES-RESONANCE ENHANCED HARMONICS
IN LOW PRESSURE CAPACITIVE DISCHARGES

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INTRODUCTION — STANDING WAVES
(Lieberman et al, PSST, 2002)

- Cylindrical discharge driven at outer radius
- Linear sheath model (constant sheath width $\bar{s}$)
- Electromagnetic fields

\[
\omega_{SW} = \left( \frac{\bar{s}}{l} \right)^{1/2} \frac{2.405 \, c}{R}
\]

- Spatial (quarter-wave) resonance
INTRODUCTION — NONLINEAR SERIES RESONANCE
(Mussenbrock and Brinkmann, APL, 2006; Lieberman et al, PoP, 2008)

- Voltage-driven, asymmetric (single sheath) discharge
- Nonlinear (homogeneous) sheath model $\bar{s}(t) \propto \sqrt{V_s(t)}$
- Electrostatic fields

- Series resonance (capacitive sheath + inductive plasma)

$$\omega_{SR} = \left( \frac{\bar{s}}{l} \right)^{1/2} \quad \omega_p \approx N\omega$$
• Nonlinear radially-varying sheath + electromagnetic fields
• Low density regime \((E_z \gg E_r)\) with ordering

\[
\begin{align*}
  s &\ll l \ll \delta_p, R \\
\end{align*}
\]

with \(\delta_p = c/\omega_p\) the collisionless skin depth

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MODEL EQUATIONS AND BASE CASE

\[ \frac{\partial \Sigma}{\partial t} = J_d - J_{i0} + J_{e0} e^{-\Sigma^2/2\varepsilon e_0\varepsilon_0 T_e}, \quad \Sigma > 0, \quad \text{(sheath charge)} \]

\[ \frac{\partial J_d}{\partial t} = \frac{e^2 n_e}{md} (V_d - V_{dc}) - \frac{e}{2\varepsilon_0 md} \Sigma^2 - \nu J_d, \quad \text{(axial current density)} \]

\[ \frac{1}{r} \frac{\partial}{\partial r} \left( r \frac{\partial V_d}{\partial r} \right) = \mu_0 l \frac{\partial J_d}{\partial t}, \quad \text{(radial transmission line voltage)} \]

\[ \Sigma = e n_e s(r, t), \quad \nu = \text{collision frequency}, \quad V_{dc} = \text{bias voltage} \]

- Solve for \( V_d(r, t), \quad J_d(r, t), \quad s(r, t), \) and \( V_{dc} \)

- BASE CASE:
  
  \( l = 2 \text{ cm}, \quad R = 15 \text{ cm conducting electrodes} \)
  
  \( 10 \text{ mT argon}, \quad n_e = 2 \times 10^{16} \text{ m}^{-3}, \quad T_e = 3 \text{ V} \)
  
  \( V_0 = 500 \text{ V}, \quad f = 60 \text{ MHz}, \quad R_T = 0.5 \text{ } \Omega \)
- Normalized discharge voltage has weak harmonic content at $r = R$; stronger at $r = 0$
- Voltage at $r = R$ can be greater than at $r = 0$
- Normalized discharge current density shows strong harmonics; series resonance oscillations at $r = R$; strong at $r = 0$
• Maximum discharge voltage is higher at $r = 0$ than at $r = R$
• Minimum sheath width at $r = 0$ is smaller than at $r = R$
  ⇒ electrons collected near $r = 0$, ions collected near $r = R$
• Maximum sheath width at $r = 0$ is larger than at $r = R$
• Sheath motion shows series resonance oscillations; strong at $r = 0$
• Fundamental voltage and current show a weak standing wave
• Voltage shows significant 3rd and 4th harmonics
• Discharge current shows strong central \((r = 0)\) 3–6 harmonics
  — \(J \propto \omega CV\) (capacitive sheath)
  — Series resonance enhancement (sheath resonates with plasma)
- Total electron power has strong 3rd and 4th harmonics
- Excitation voltage is nearly sinusoidal with a dc bias
- Excitation current has strong harmonic content
- Strong spatial resonance effects seen near $M\omega = \omega_{SW}$
CENTER-TO-AVERAGE POWER/AREA

\[ n_e = 2 \times 10^{16} \text{ m}^{-3} \]

\[ n_e = 5 \times 10^{15} \text{ m}^{-3} \]

- Dashed vertical lines are the spatial resonances
- Even at lower frequencies there is a significant center-peaking of the electron power/area
CONCLUSIONS

• We developed and numerically solved a nonlinear radial transmission line model of an asymmetrically driven rf capacitive discharge.

• We found that the series resonance-enhanced harmonics of the driving frequency coupled strongly to the standing wave spatial resonances.

• We found significant center-peaking of the electron power/area, even at low excitation frequencies.

• These phenomena may be responsible for the center-peaked plasma density seen experimentally in high frequency capacitive discharges (e.g., Sawada et al, JJAP, 2014).

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