FERMI ACCELERATION REVISITED —

FROM COSMIC RAYS TO DISCHARGE HEATING

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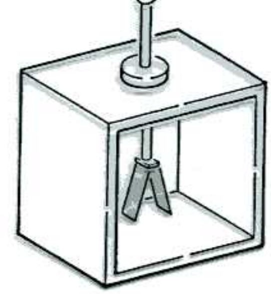
SUMMARY OF TALK

- Cosmic Rays
- Dynamics
- ECR Discharges
- Capacitive RF Discharges
- Inductive RF Discharges

COSMIC RAYS

DISCOVERY



1895: W.C. Roentgen, X-rays (1902)

1902: C.T.R. Wilson, Cloud chamber (1927)

C.T.R. Wilson at Sydney Sussex College in Cambridge, England, had noticed earlier that even in a well-shielded ionization chamber a certain electrical leakage always occurred due to the production of ions in the chamber. Radioactive substances and X rays would have been stopped by the shields placed around the chamber. Therefore, Wilson theorized, some source of residual ionization must exist that could penetrate a great thickness of material.

Wilson suspected that the radiation might be cosmic. He set up his apparatus at Peebles in Scotland in a Caledonian Railway tunnel and found the same discharge rate outside and inside the tunnel. He concluded:

> There is thus no evidence of any falling off of the rate of production of ions in the vessel, although there were many feet of rock overhead. It is unlikely, therefore, that the ionisation is due to radiation which has traversed our atmosphere. . . .[203]

1912: V. Hess, Cosmic rays (1936)

At six o'clock on the morning of August 7, 1912, the Austrian physicist Viktor Hess and two companions climbed into a balloon gondola for the last of a series of seven launches. The flight, which had started at Aussig on the Elbe, was under the command of Captain W. Hoffory. The meteorological observer was W. Wolf, and Hess listed himself as "observer for atmospheric electricity." Over the next three or four hours the balloon rose to an altitude above 5 kilometers, and by noon the group was landing at Pieskow, some 50 kilometers from Berlin. During the six hours of flight Hess had carefully recorded the readings of three electroscopes he used to measure the intensity of radiation and had noted a rise in the radiation level as the balloon rose in altitude.

In the *Physikalische Zeitschrift* of November 1 that year Hess wrote, "The results of these observations seem best explained by a radiation of great penetrating power entering our atmosphere from above. . . ."[3] This was the beginning of cosmic-ray astronomy. Twenty-four years later Hess shared the Nobel Prize in physics for his discovery.

POSING IN HIS BALLOON

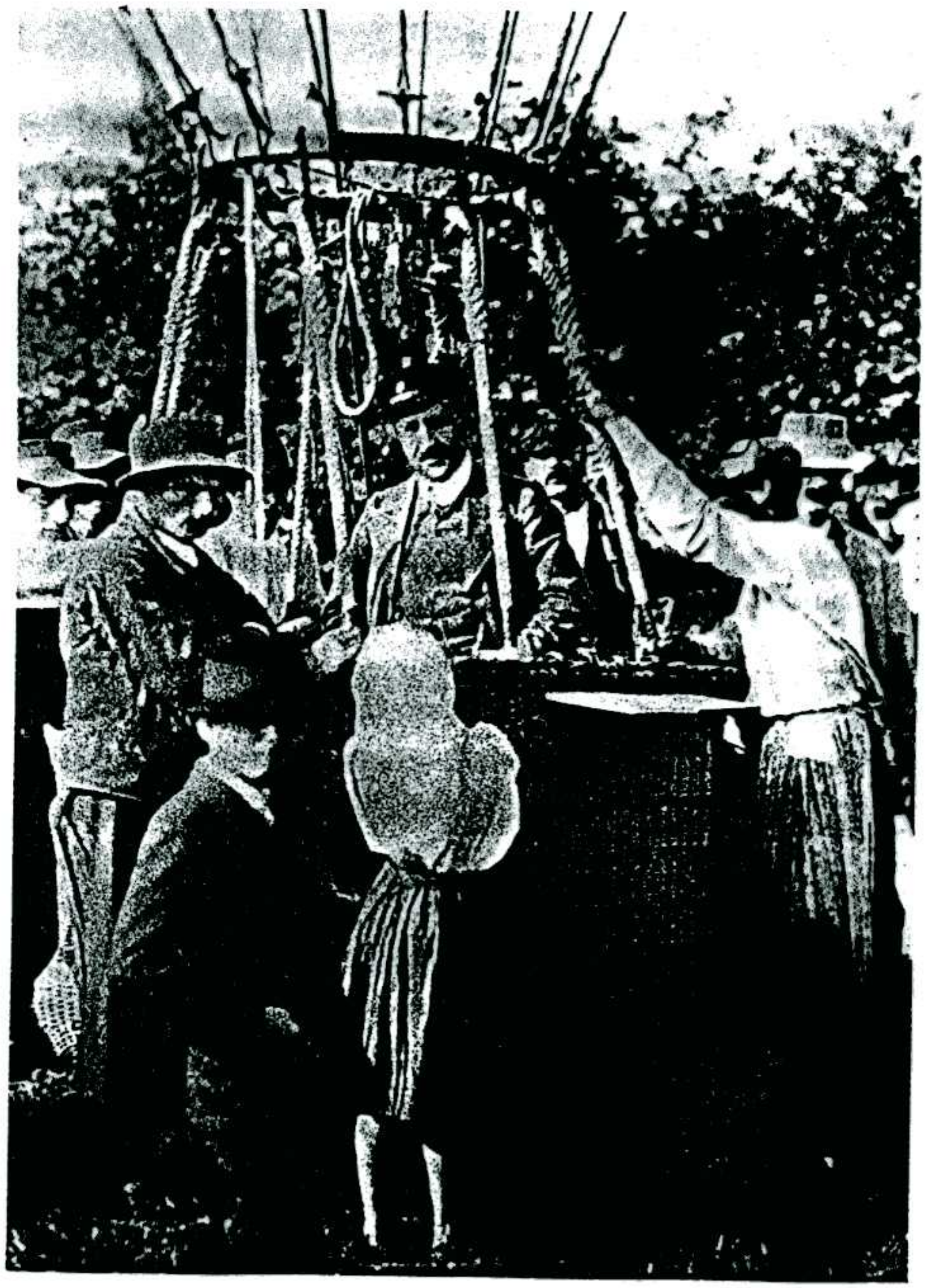


Figure 1.3
Victor Hess, discoverer of the cosmic rays, after his 1912 balloon flight that reached an altitude of 17,500 feet. (Photograph courtesy of Martin A. Pomerantz.)

ORIGIN

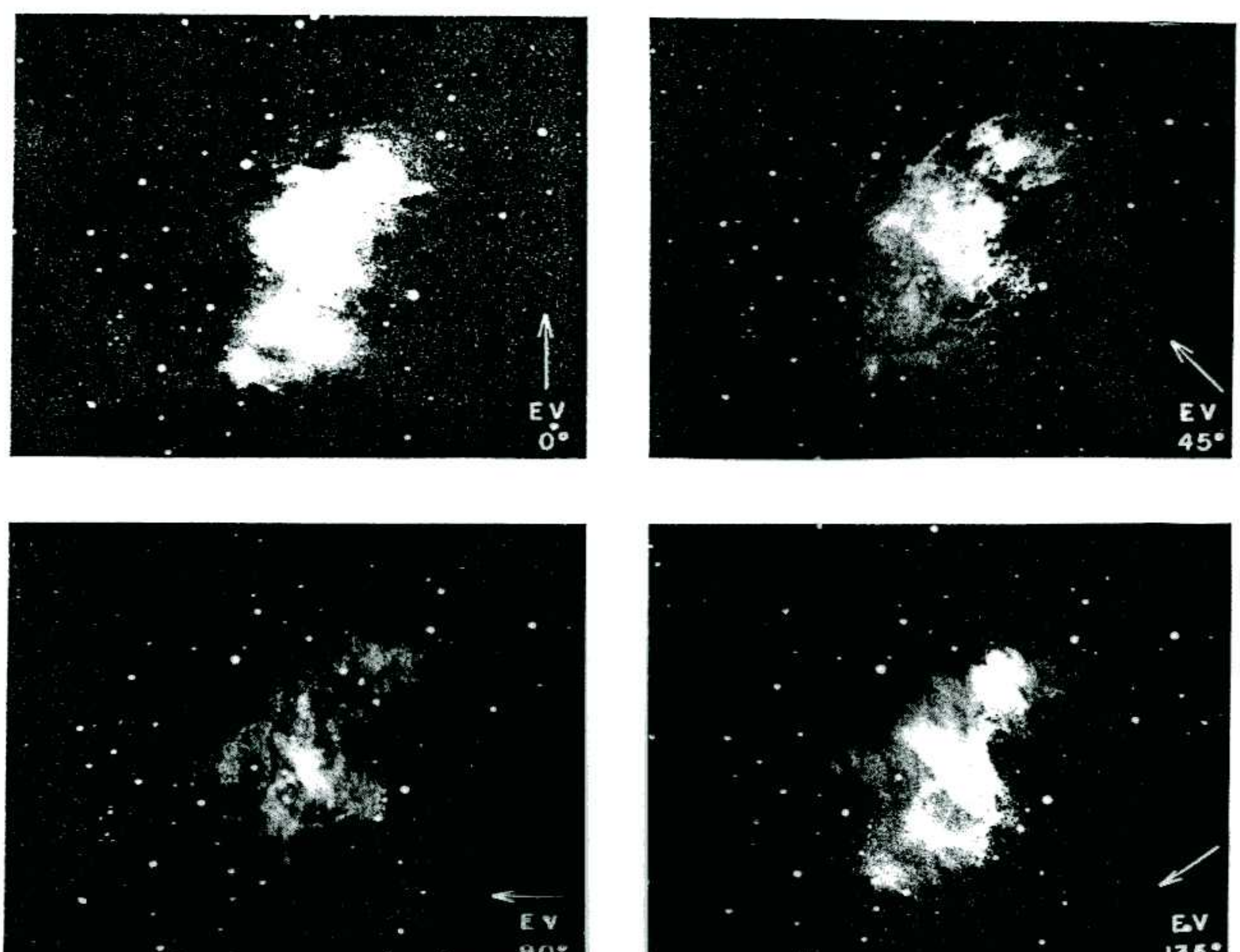
- Supernovas
- Energy balance:

$$\frac{10^{43}\ \mathrm{J}}{50\ \mathrm{yrs}} \approx \frac{e \times 1\ \mathrm{eV/cm^3} \times 10^{67}\ \mathrm{cm^3}}{10^7\ \mathrm{yrs}}$$

- Fast particles:

1054: Crab nebula

1955: Synchrotron radiation ($W_e > 10^{11}$ eV)



PROPOSED ACCELERATION MECHANISMS

- High voltages
- Shock waves
- Moving magnetic fields (Fermi, 1949)

PHYSICAL REVIEW VOLUME 75, NUMBER 8 APRIL 15, 1949

On the Origin of the Cosmic Radiation

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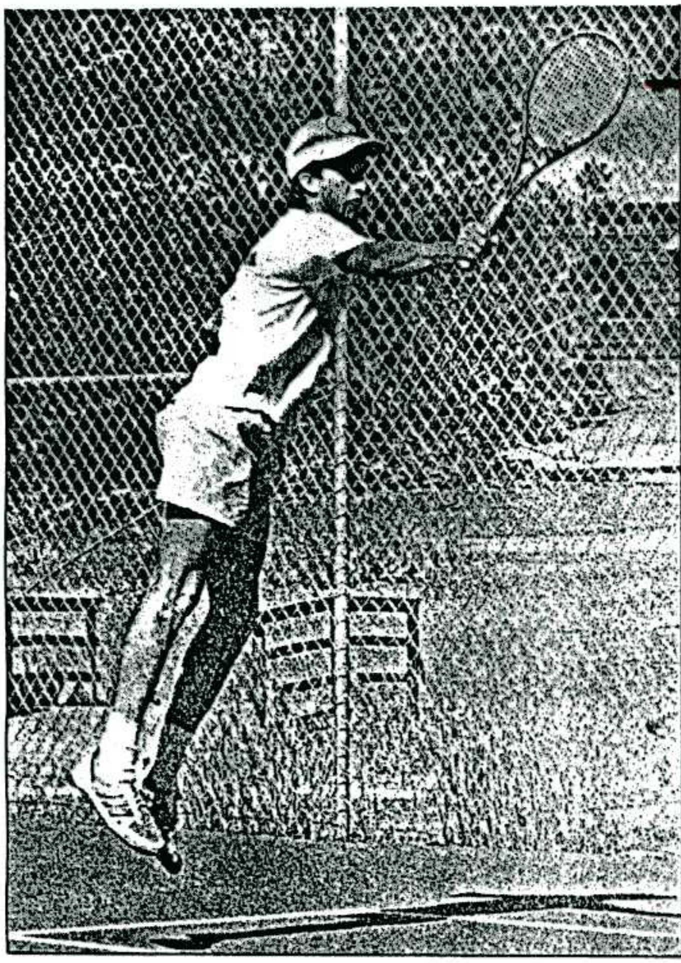
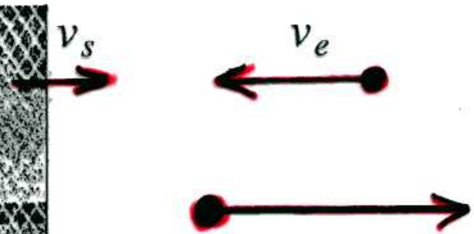
(Received January 3, 1949)

A theory of the origin of cosmic radiation is proposed according to which cosmic rays are originated and accelerated primarily in the interstellar space of the galaxy by collisions against moving magnetic fields. One of the features of the theory is that it yields naturally an inverse power law for the spectral distribution of the cosmic rays. The chief difficulty is that it fails to explain in a straightforward way the heavy nuclei observed in the primary radiation.

It may happen that a region of high field intensity moves toward the cosmic-ray particle which collides against it. In this case, the particle will gain energy in the collision. Conversely, it may happen that the region of high field intensity moves away from the particle. Since the particle is much faster, it will overtake the irregularity of the field and be reflected backwards, in this case with loss of energy. The net result will be an average gain, primarily for the reason that head-on collisions are more frequent than overtaking collisions because the relative velocity is larger in the former case.

STOCHASTIC HEATING





$v'_e = v_e + 2v_s$

$v_s > 0$, energy gain

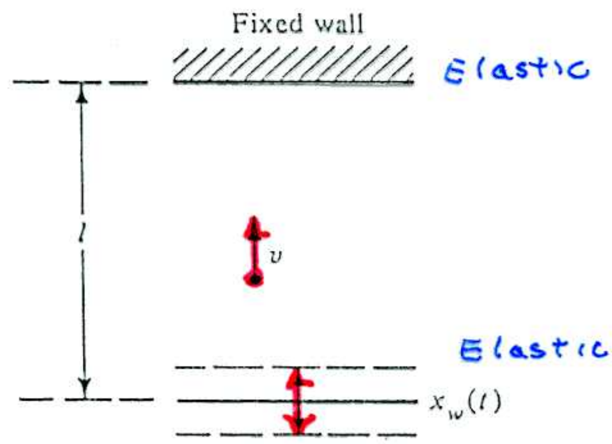
$v_s < 0$, energy loss

- Let $v_s = v_{s0} \cos \omega t$

  Then $< v'^2_e > = v_e^2 + 2v_{s0}^2$ corresponds to a net energy gain.

DYNAMICS

FERMI MAPS—A PARADIGM IN DYNAMICS



- Simplified Fermi map (Lieberman and Lichtenberg, 1972)

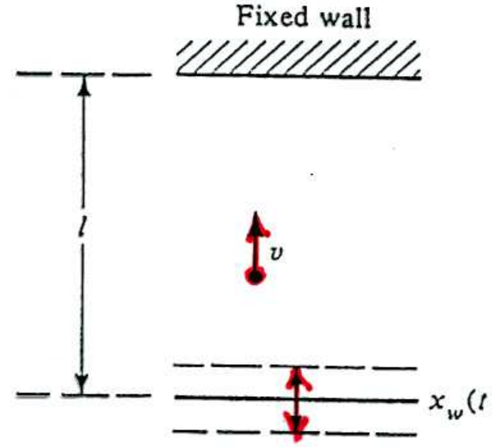
  Oscillating wall imparts momentum to ball without physically changing its position:

$$u_{n+1} = |u_n + \sin\psi_n|$$

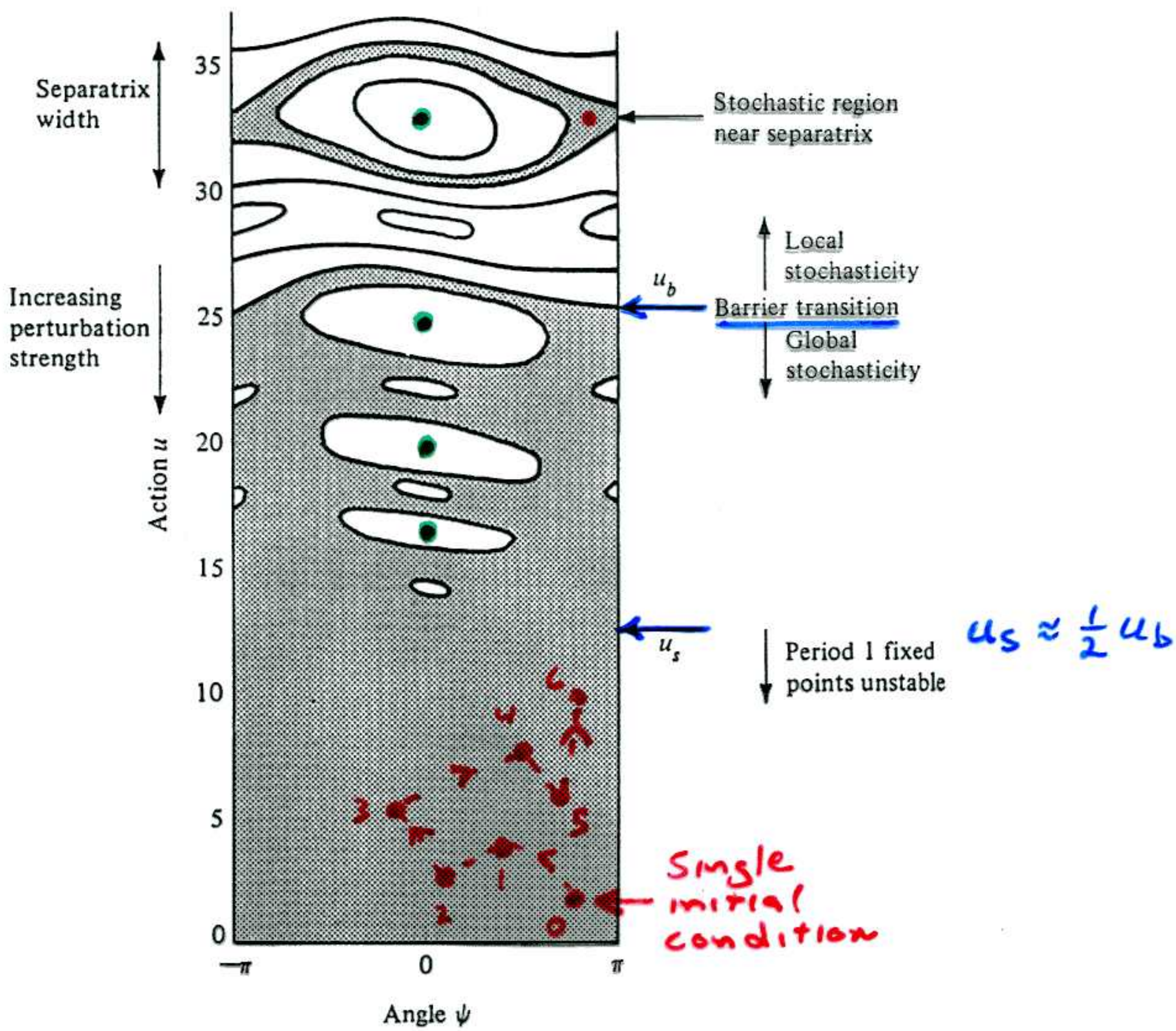
$$\psi_{n+1} = \psi_n + \frac{2\pi M}{u_{n+1}}$$

$u_n = v_n/2\omega a$, $M = l/2\pi a$, $\psi_n = \omega t_n$

SIMPLIFIED FERMI MAP



- $$u_{n+1} = |u_n + \sin\psi_n|$$
  $$\psi_{n+1} = \psi_n + \frac{2\pi M}{u_{n+1}}$$
- $< u_{n+1}^2 >_\psi - < u_n^2 >_\psi = \frac{1}{2} \Rightarrow$ net heating



LOCAL APPROXIMATION BY STANDARD MAP

$$K = 2\pi M/u^2$$

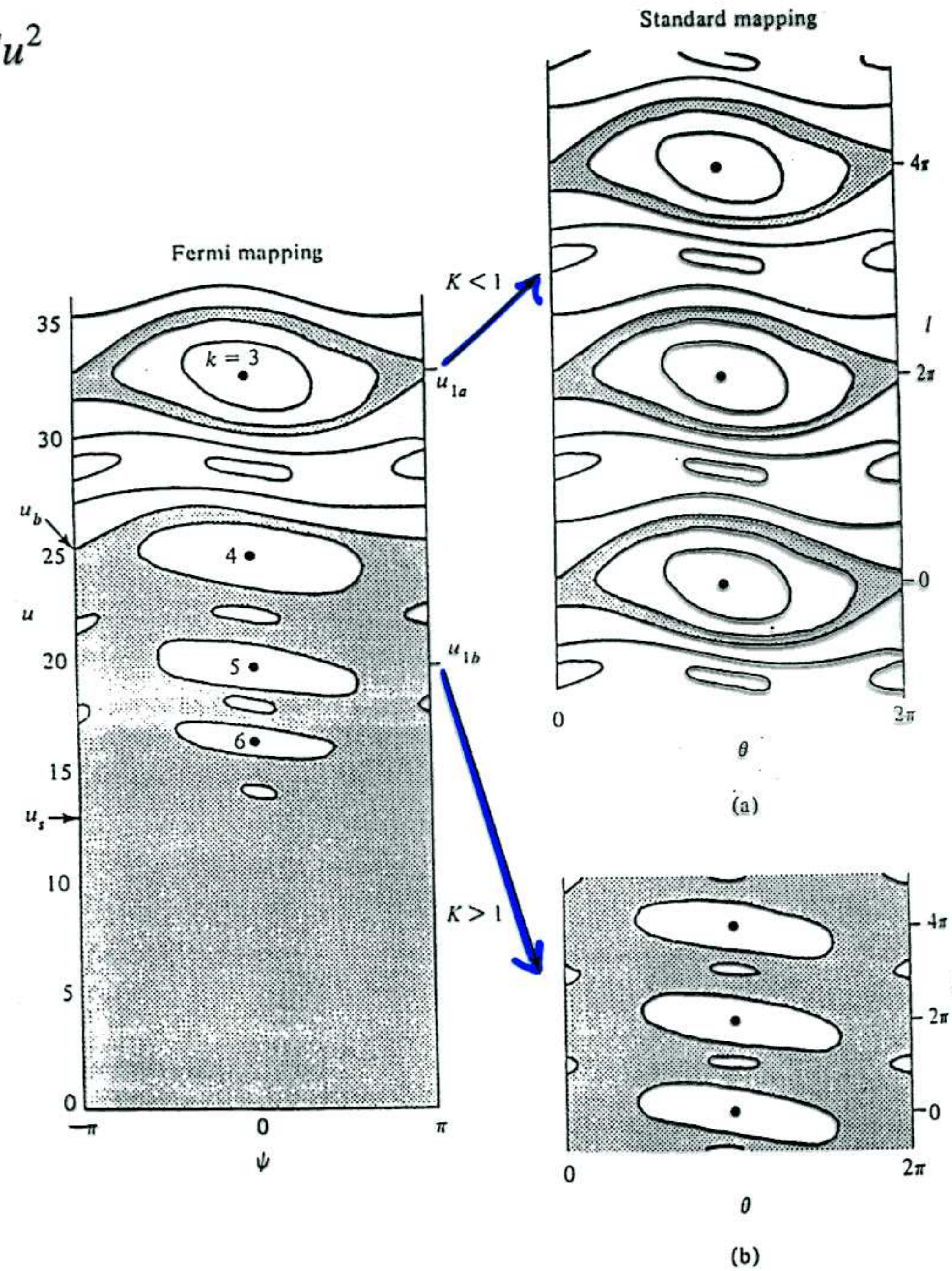


Figure 4.2. Local approximation of the Fermi mapping by the standard mapping. (a) Linearization about u_{1a} leading to K small and local stochasticity; (b) linearization about u_{1b} leading to K large and global stochasticity.

FOKKER-PLANCK EQUATION AND DIFFUSION

- Consider perturbed twist map ($I \equiv u$; $\theta \equiv \psi$)

$$I_{n+1} = I_n + \varepsilon F(I_{n+1}, \theta_n)$$

$F \equiv \sin \theta_n$

- Assume change in I is small on time scale (n) over which θ randomizes

- Let $P(I, n)$ be the distribution in action alone (average over phases):

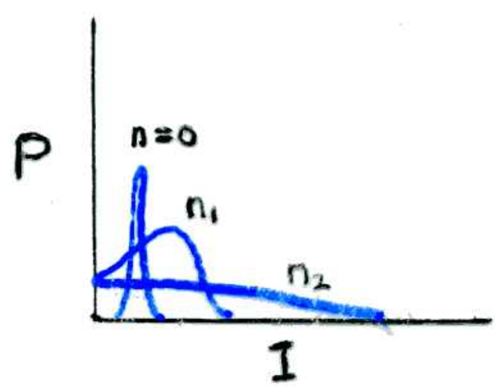
$$\frac{\partial P}{\partial n} = \frac{1}{2} \frac{\partial}{\partial I}\left(D \frac{\partial P}{\partial I}\right)$$ Fokker-Planck equation

- $D(I)$ is the local diffusion coefficient

- If the phases are uncorrelated over a single step:

$$D_{QL} = (2\pi)^{-1} \int_0^{2\pi} (\Delta I)^2 \, d\theta$$

- For correlated phases, obtain corrections to quasilinear diffusion



STEADY STATE SOLUTIONS

- $$\frac{\partial}{\partial I}\left(D\,\frac{\partial P}{\partial I}\right)=0$$

- Assume zero flux:

$$D\,\frac{\partial P}{\partial I}=0 \quad \Rightarrow \quad P=\text{const}$$

- Simplified map:

$$D_{QL}=\frac{1}{2};\;\; P(u)=\text{const}$$

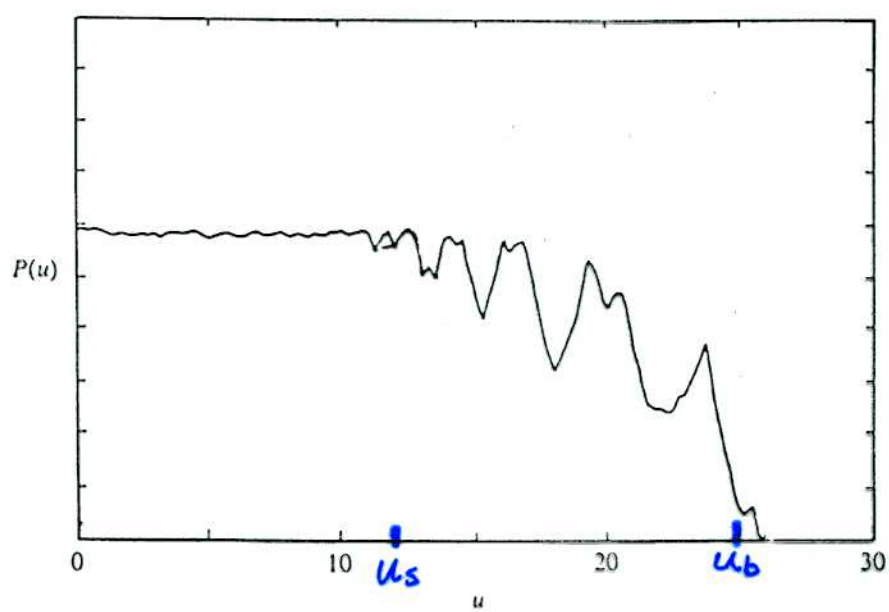


Figure 5.11. Comparison of velocity distribution $P(u)$ for the simplified Fermi map

CAPACITIVE RF DISCHARGE

ORIGINS

SOVIET PHYSICS – TECHNICAL PHYSICS VOL. 16, NO. 7 JANUARY, 1972

STATISTICAL HEATING OF ELECTRONS AT AN OSCILLATING PLASMA BOUNDARY

V. A. Godyak

Translated from Zhurnal Tekhnicheskoi Fiziki, Vol. 41, No. 7, pp. 1364-1368, July, 1971
Original article submitted September 1, 1970

In an oscillating double sheath, the potential distribution, and thus the coordinate of the electron-reflection point depend on the time, and the electron reflection is analogous to that of solid particles from a vibrating wall. On the average particles acquire energy in this case (the Fermi acceleration mechanism).

Stochastic plasma heating by rf fields

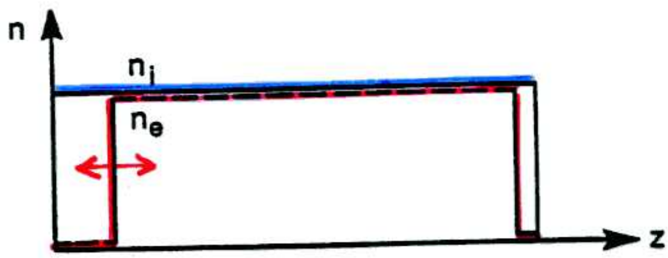
A. I. Akhiezer and A. S. Bakai

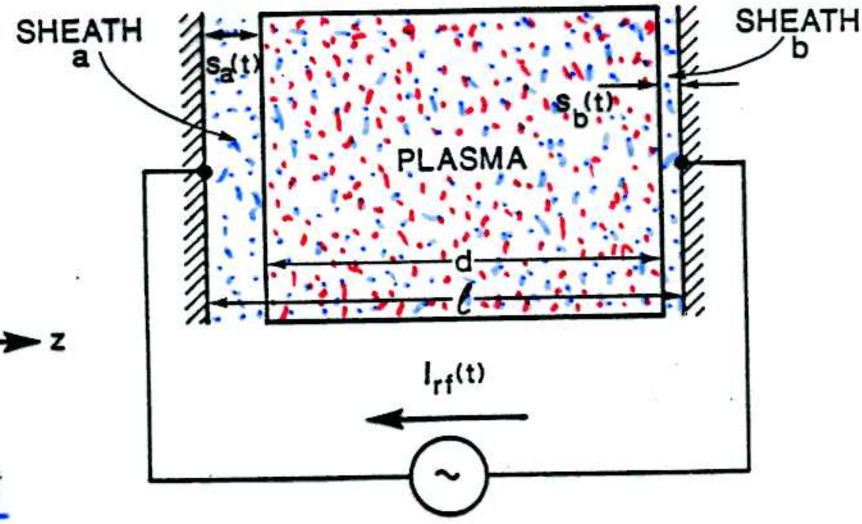
Physicotechnical Institute, Academy of Sciences of the Ukrainian SSR

(Submitted July 1, 1975; resubmitted January 9, 1976)
Fiz. Plazmy 2, 654–657 (July–August 1976)

A study is made of the possibility of plasma heating by regular rf fields concentrated near the plasma boundary or in thin layers within the plasma. The proposed heating method is based on Fermi stochastic acceleration. Under certain conditions the average particle energy increases in proportion to the square of the time, and a steady-state step-function particle velocity distribution is established in a comparatively short time.

HOMOGENEOUS MODEL





- $J(t) = env_s(t) = en\tilde{v}_s \cos\omega t$

  $v' = v + 2\tilde{v}_s \cos\omega t$

  $\langle \Delta W \rangle_t = \langle \frac{1}{2} m(v'^2 - v^2) \rangle_t = m\tilde{v}_s^2$

- $$S_{\text{stoc}} = \langle n \int_{v_s}^{\infty} (v - v_s) \underbrace{\frac{1}{2} m(4vv_s + 4v_s^2)}_{\Delta W} f_e(v)\, dv \rangle_t$$

  $S_{\text{stoc}} = \frac{1}{2} mn\tilde{v}_s^2 \bar{v}_e$, where $\bar{v}_e = (8kT_e/\pi m)^{1/2}$

- $S_{\text{ohm}} = \frac{1}{2} \frac{\tilde{J}^2}{\sigma} d$, where $\sigma = e^2 n/m\nu_m$

  $S_{\text{ohm}} = \frac{1}{2} mn\tilde{v}_s^2 \nu_m d$

  $\nu_m \propto$ gas pressure (electron-neutral collision frequency)

- $S_e = \frac{1}{2} mn\tilde{v}_s^2(\nu_m d + 2\bar{v}_e)$

  $\nu_{\text{eff}} = \nu_m + 2\bar{v}_e/d$

  Ohmic (ν_m) Stochastic ($2\bar{v}_e/d$)

EXPERIMENTAL EVIDENCE

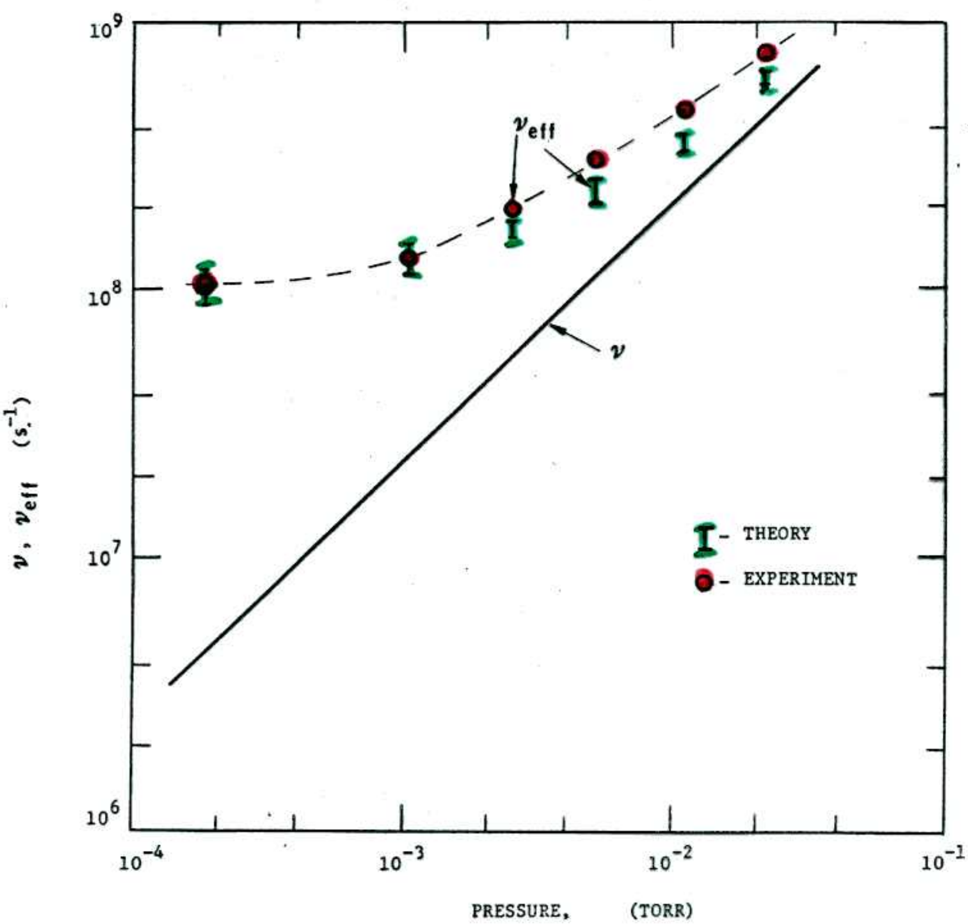


FIG. 2. Electron effective collision frequency ν_{eff} (theory and experiment) and electron–neutral-atom collision frequency ν as functions of mercury-vapor pressure p.

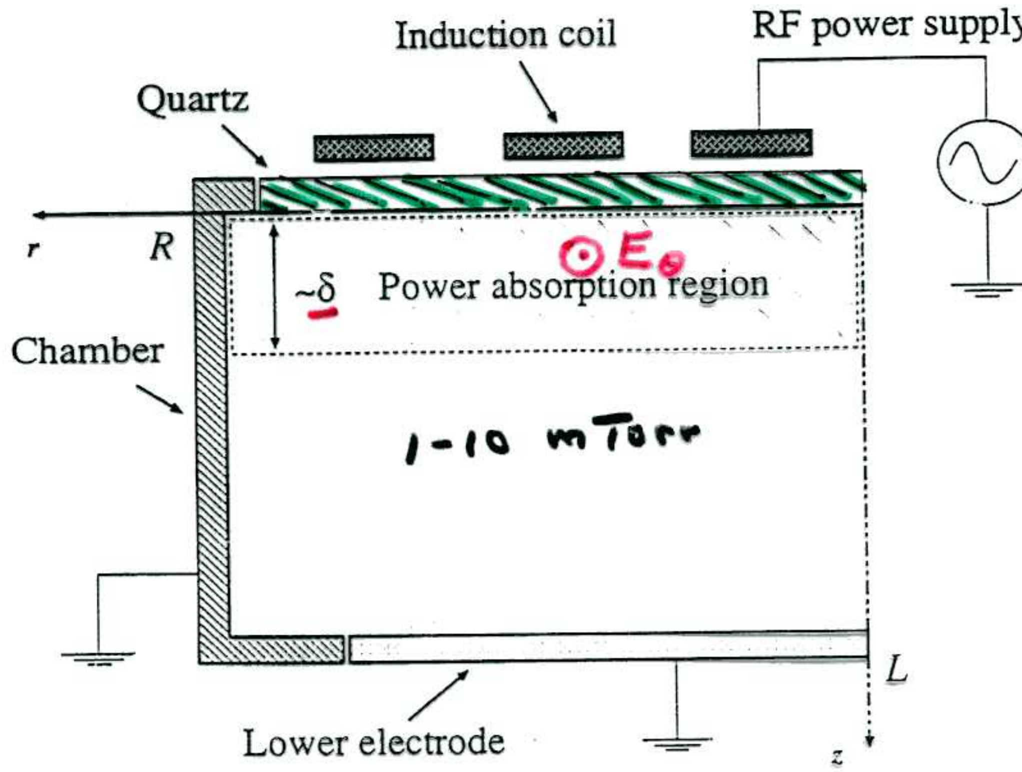
J. Appl. Phys., Vol. 57, No. 1, 1 January 1985

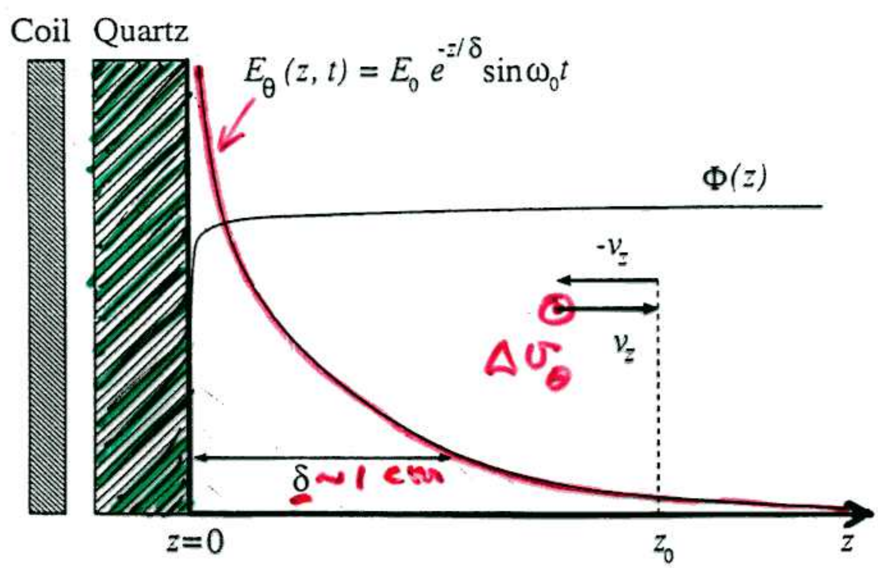
O. A. Popov and V. A. Godyak

(Godyak, Piejak, Alexandrovich, 1992)

INDUCTIVE RF DISCHARGES

INDUCTIVE RF DISCHARGE





SIMPLE MODEL

- $E_\theta(z,t) = E_0 \mathrm{e}^{-\alpha z} \cos(\omega t + \phi)$

  $z(t) = -v_z t, \qquad t < 0$

  $z(t) = v_z t, \qquad t > 0$

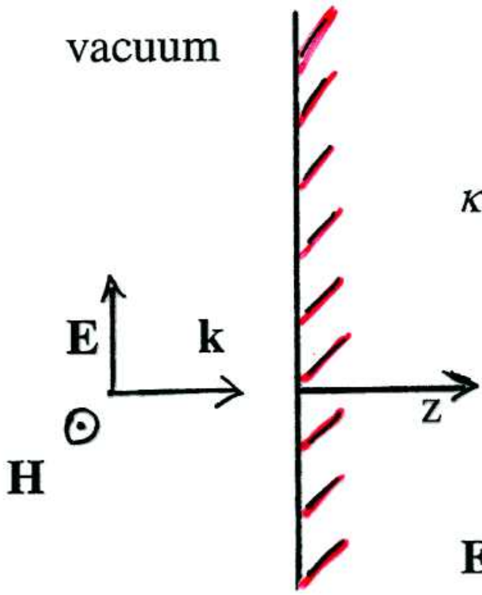
- $\Delta v_\theta(\phi) = \int_{-\infty}^{\infty} dt \, \frac{e}{m} \, E_\theta((z(t), t)$

- $\Delta W(v_z) = \frac{1}{2} m < (\Delta v_\theta)^2 >_\phi$

- $S_{\mathrm{stoc}} = n \int_0^{\infty} v_z \, \Delta W \, f_e(v_z) \, dv_z$

- Full range of collisionality
  (Vahedi et al, 1995)

SKIN DEPTHS



$$\kappa_p = 1 - \frac{\omega_p^2}{\omega(\omega - j\nu_m)}$$ (plasma dielectric constant)

$$\omega_p^2 = \frac{e^2 n}{\epsilon_0 m}$$ (plasma frequency)

$$\mathbf{E} \propto e^{-\alpha z - j\beta z}$$

- Local Theory

$$\alpha = \frac{\omega}{c} \operatorname{Im} \kappa_p^{1/2} \equiv \frac{1}{\delta}$$

(a) Collisionless ($\nu_m \ll \omega$) $\Rightarrow$ $\delta_p = \dfrac{c}{\omega_p}$ ~ 1 cm

(b) Collisional ($\nu_m \gg \omega$) $\Rightarrow$ $\delta_c = \dfrac{c}{\omega_p}\left(\dfrac{2\nu_m}{\omega}\right)^{1/2} = \left(\dfrac{2}{\omega \mu_0 \sigma}\right)^{1/2}$ ~ 1 cm

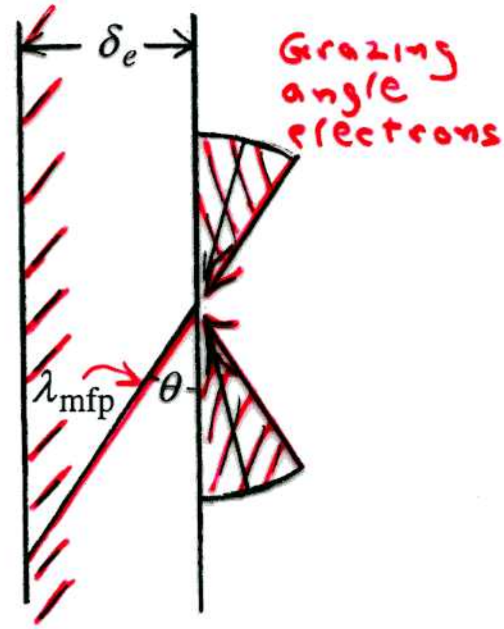
- Nonlocal Theory

(c) Anomalous ($\bar{v}_e/\delta_e \gg \omega, \nu_m$) $\Rightarrow$ $\delta_e \approx \dfrac{c}{\omega_p}\left(\dfrac{2\bar{v}_e \omega_p}{\omega c}\right)^{1/2}$ ~ 1 cm

- However, the E-field decay is not exponential

ANOMALOUS SKIN DEPTH

$\lambda_{\mathrm{mfp}} \gg \delta_e$



- Consider only electrons having incident angles such that at least one mean free path lies within a skin depth

$$\theta \approx \frac{\delta_e}{\lambda_{\mathrm{mfp}}}$$

$$n_{\mathrm{eff}} \approx n\,\frac{2\pi\theta}{2\pi} \approx n\,\frac{\delta_e}{\lambda_{\mathrm{mfp}}}$$

$$\sigma_{\mathrm{eff}} \approx \frac{e^2 n_{eff}}{m\nu_m} \approx \frac{e^2 n\delta_e}{m\bar{v}_e}$$

Set $\delta_e = \left(\dfrac{2}{\omega\mu_0\sigma_{\mathrm{eff}}}\right)^{1/2}$ to obtain δ_e

- For many inductive discharges, $\delta_p \approx \delta_c \approx \delta_e$ ~ 1 cm

ANOMALOUS HEATING THEORY AND EXPERIMENTS

- Mechanism observed and anomalous skin effect suggested (Godyak, Piejak, and Alexandrovich, 1994)
- Collisionless theory and PIC simulation (Turner, 1993)
- Full range of electron collisionality (Shaing, 1993)
- Theory RF magnetic field effects (Cohen and Rognlien, 1994)
- Observation of negative electron power absorption (Godyak and Kolobov, 1997)
- Observation of RF magnetic field effects (Godyak, Piejak, and Alexandrovich, 1999)

- First plasma experiment (Demirkhanov et al, 1964)
- First plasma kinetic theory (Weibel, 1967)
- First finite length plasma theory (Blevin et al, 1970)
- Theory of RF magnetic field effects (Blevin and Thonemann, 1962)
- RF magnetic fields (Rotomak experiment) (Jones, 1980)

- First experiment on anomalous skin effect in metals (Pippard, 1949)
- First kinetic theory of anomalous skin effect (Reuther and Sondheimer, 1949)

PIPPARD'S WORK

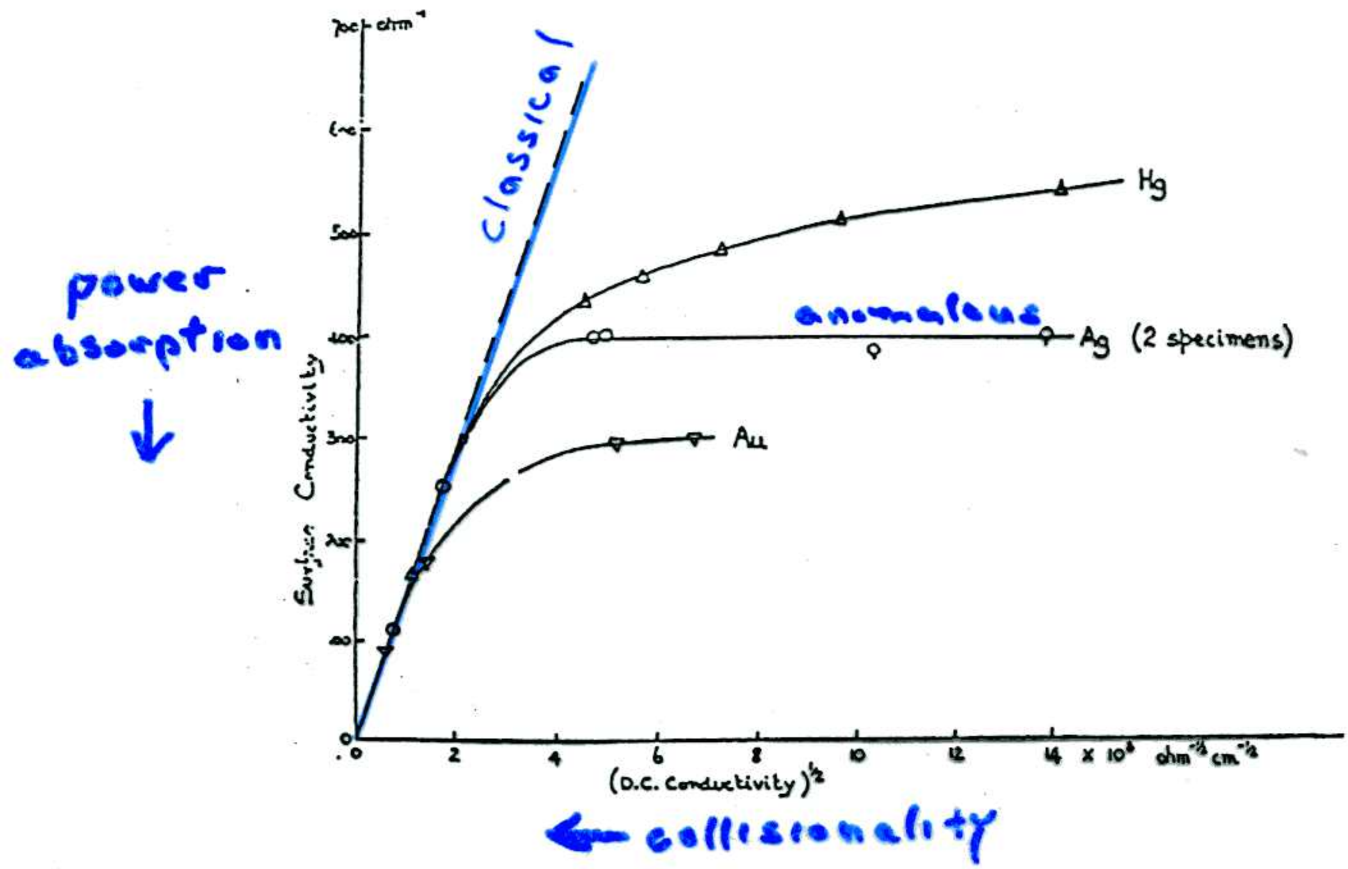
Physica XV, no 1--2 April 1949

THE HIGH FREQUENCY SKIN RESISTANCE OF METALS AT LOW TEMPERATURES

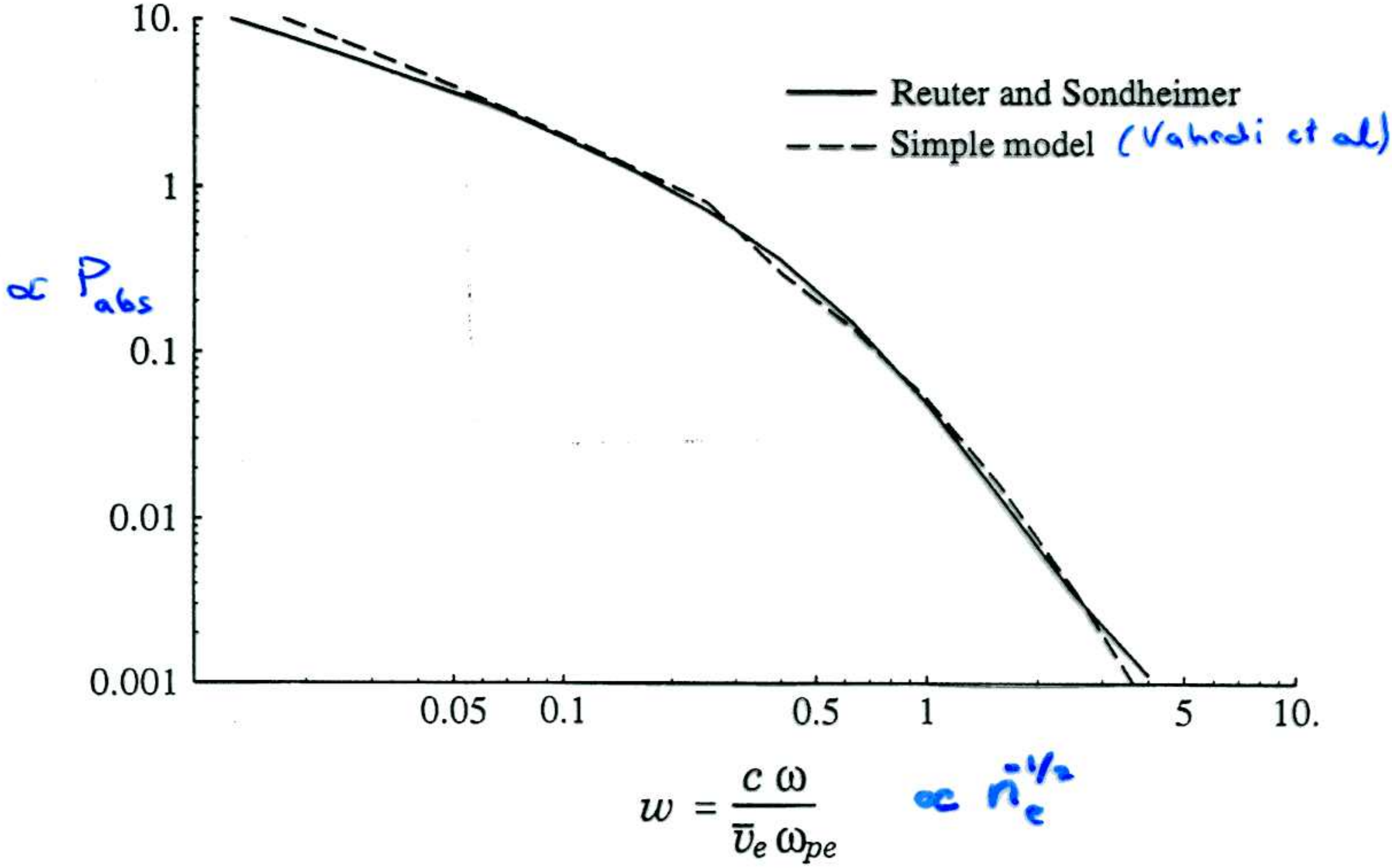
by A. B. PIPPARD

Royal Society Mond Laboratory Cambridge, England

During the course of experiments, just before the war, on the resistance of superconducting tin at a frequency of 1500 Mc/s, H. London [1]) observed that the measured values of the resistance of the normal metal above the transition temperature (3.7°K) disagreed markedly with the values predicted by the classical theory of the skin effect, and tentatively suggested that the long free path of the electrons at low temperatures was the cause of the anomaly.



COMPARISON OF RESULTS



Comparison of the normalized input power. Solid line is obtained from the classical non-local theory of Reuter and Sondheimer (1948) (with $\nu_{en}/\omega = 0.008$); dashed line is the simple collisionless model.

CONCLUDING REMARKS

- Collisionless electron heating, which is usually associated with high temperature space and fusion plasmas, is a fundamental process in the warm, low pressure RF and microwave discharges used in today's industrial plasma technology

- Such heating is due to electron interaction with localized electric fields, which leads to randomization of the electron phase even in the absence of collisions

- The Fermi acceleration model of a ball bouncing between a fixed and an oscillating wall is a useful model to describe such heating

(For further information see the review article by M.A. Lieberman and V.A. Godyak, *IEEE Trans. Plasma Sci.*, **26**, 955–986, June 1998)