On a Network Creation Game

CS294-4 Presentation

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Slides borrowed from Alex Fabrikant

Paper Overview

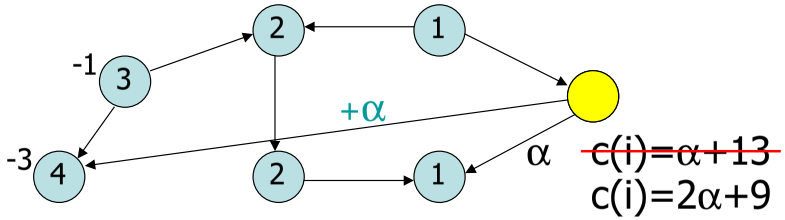
- Study the Internet using game theory
- Define a model for how connections are established
- Compute the "price of anarchy" within the model

Game Theoretical Model

- N players
- Each buys an undirected link to a set of others (s_i)
- Combine all these links to form G
- Anyone can use the link paid for by i
- Cost to player:

$$c(i) = \alpha \cdot |s_i| + \sum_j d_G(i,j)$$

Example



(Convention: arrow *from* the node buying the link)

Model Limitations

- Each link paid for by single player
- Disproportionate incentive to keep graph connected
- Hop count is only metric
  - All links cost the same
  - No handling of congestion, fault-tolerance
- Reaching each node equally as valuable

Social Cost

- Social cost is sum of all the per-player costs c(i)
- There is an optimal graph G resulting in lowest social cost
  - Best graph overall
  - But not necessarily best for all (or any players)
  - Hence, rational players may deviate from global optimum

Nash Equilibrium

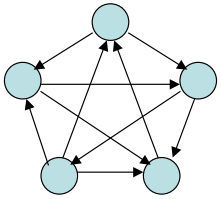
- Nash Equilibrium: no single player can make a unilateral change that will him
  - Rational players will maintain a nash equilibrium
- Don’t always exist
  - They do in this model
- Are not always achievable through rational actions

Price of Anarchy

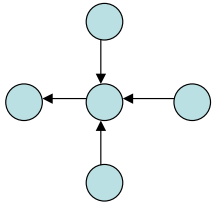
- Ratio between the social cost of a worst-case Nash equilibrium and the optimum social cost
- Goal: compute bounds on the price of anarchy

Social optima

- $\alpha<2$: clique
- any missing edge can be added at cost α and subtract at least 2 from social cost

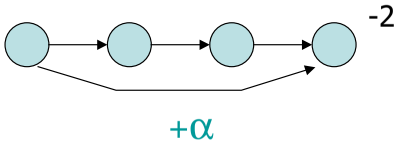


- $\alpha\geq 2$: star
- Any extra edges are too expensive.



Nash Equilibria

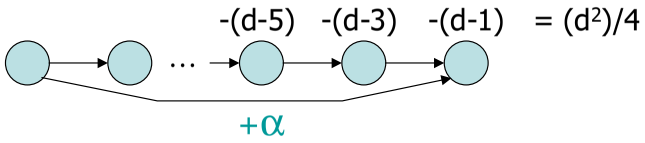
- For $\alpha<1$, Nash equilibrium is complete graph
- For $1<\alpha<2$, Nash equilibrium graph has to be of diameter at most 2.



- Hence worst equilibrium is a star

General Upper Bound

- Assume $\alpha>2$ (the interesting case)
- Lemma: if G is a N.E., $d_G(i,j) < 2\sqrt{\alpha}$
  - Generalization of the above:



General Upper Bound (cont.)

- A counting argument then shows that for every edge present in a Nash equilibrium, $\sqrt{\alpha}$ others are absent
- Then:

$$\begin{aligned} C(G) &\leq \alpha \cdot O(n^2/\sqrt{\alpha}) + n(n-1)\cdot 2\sqrt{\alpha} \\ &= O(\sqrt{\alpha}n^2) \end{aligned}$$

Complete Trees

- A complete k-ary tree of depth d, at $\alpha=(d-1)n$, is a Nash equilibrium
- Can't drop any links (infinite cost increase)
- Any new edge has to improve distance to each node by (d-1) on average
- Lower bound: price of anarchy approaches 3 for large d,k

Tree Conjecture

- Experimentally, all nash equilibria are trees for sufficiently large α
- If this is the case, can compute much better upper bound: 5
- Proof relies on having a "center node" in graph

Discussion

- Is 5 an acceptable price of anarchy?
  - If not, what can we do about it
- A center node is a *terrible* topology for the Internet

Getting back to P2P

- Game theory and Nash equilibria important to P2P networks
  - Incentive to cooperate
- What about the network model?
  - In some networks, edges are directed (e.g. Chord)
  - Extra routing constraint
  - Incomplete information

Chord Example

- Assume successor links are free
- Is there an α for which Chord is a Nash equilibrium?
- Short hops aren't worth it except for very small α
- For large α (>n), defecting and maintaining only a link to your successor is a win

Discussion?